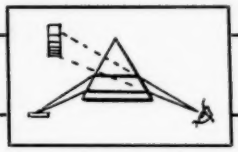
Journal of the OPTICAL SOCIETY *of* AMERICA

VOLUME 23
No. 2



FEBRUARY
1933

CONTENTS

Saturation Scale for Yellow Colors . . . . . DEANE B. JUDD 35

Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight . . . . IRWIN G. PRIEST 41

Color Filters for Altering Color Temperature. Pyrometer Absorption and Daylite Glasses . . . . . H. P. GAGE 46

Line Intensity and Energy Distribution in High and Low-Pressure Mercury Arcs . . . . . L. B. JOHNSON AND L. BURNS 55

Measurement of Erythemal Ultraviolet Radiation . . . . A. H. TAYLOR 60

Herschel Effect as a Result of Simultaneous Forward and Reverse Reactions . . . . . KARL M. JOEHNCK AND JULIAN M. BLAIR 67

Relation between the Resolving Power of a Spectroscope and the Principle of Uncertainty . . . . . STANFORD GOLDMAN 70

Published monthly by the

AMERICAN INSTITUTE OF PHYSICS
Incorporated
11 East 38th Street, New York, N. Y.

FOR AND UNDER THE EDITORIAL SUPERVISION OF

THE OPTICAL SOCIETY OF AMERICA

THE OPTICAL SOCIETY OF AMERICA

OFFICERS OF THE SOCIETY

E. C. CRITTENDEN, *President*
Bureau of Standards
Washington, D. C.

W. B. RAYTON, *Vice-President*
Bausch & Lomb Optical Company
Rochester, New York

L. B. TUCKERMAN, *Secretary*
Bureau of Standards
Washington, D. C.

HENRY F. KURTZ, *Treasurer*
Bausch & Lomb Optical Company
Rochester, New York

The Council consists of the above officers; the Editor of the Journal; the past President; and four elected members:

L. A. JONES, *Past President*
Eastman Kodak Company
Rochester, New York

F. K. RICHTMYER, *Editor*
Cornell University
Ithaca, New York

Elected Members

G. S. FULCHER
Administration Building
World's Fair
Chicago, Illinois

A. C. HARDY
Massachusetts Institute of Technology
Cambridge, Massachusetts

T. R. WILKINS
University of Rochester
Rochester, New York

A. G. WORTHING
University of Pittsburgh
Pittsburgh, Pennsylvania

THE OPTICAL SOCIETY OF AMERICA was organized in 1916 "*to increase and diffuse the knowledge of optics in all its branches, pure and applied, to promote the mutual interests of investigators of optical problems, of designers, manufacturers and users of optical instruments and apparatus of all kinds, and to encourage cooperation among them.*" It aims to serve the interests of all who are engaged in any branch of optics from fundamental research to the manufacture of optical goods.

MEMBERSHIP

Regular Members: "Any person who has contributed materially to the advancement of optics is eligible for regular membership."

Associate Members: "Any person who is interested in optics is eligible for associate membership."

Corporate Members: "Any corporation desirous of encouraging the work of the Society is eligible for corporation membership."

The Society cordially invites to membership all who are interested in any branch of optics, either in research, in instruction, in optical or illuminating engineering, in the manufacture and distribution of optical goods of all kinds, or in physiological and medical optics.

ANNUAL DUES

Regular members, $7.50; Associate members, $5.00; Corporate members, $50.00. These dues include subscription to the Society's official journal, the Journal of the Optical Society of America.

MEETINGS

The Society holds two meetings per year: An annual meeting in the autumn, and a midwinter meeting jointly with the American Physical Society.

For further information concerning the Optical Society of America address the Secretary, Dr. L. B. Tuckerman, Bureau of Standards, Washington, D. C.

ROCHESTER, NEW YORK SECTION OF THE OPTICAL SOCIETY OF AMERICA

R. KINGSLAKE, *President*
E. C. FRITTS, *Secretary*

CLIFTON TUTTLE, *Vice-President*
A. A. LEVIN, *Treasurer*

Councilors: C. E. BAHN, WALTER CLARK, BRIAN O'BRIEN, A. L. SCHOEN, T. R. WILKINS.

Journal of the
OPTICAL SOCIETY *of* AMERICA

VOLUME 23 FEBRUARY, 1933 NUMBER 2

Saturation Scale for Yellow Colors*

DEANE B. JUDD, *Bureau of Standards, Washington, D. C.*
(Received December 5, 1932)

The relationship between colorimetric purity of the stimulus and saturation of the color evoked under specified observing conditions has been determined for yellow colors by using the yellow series of Lovibond glasses (dominant wave-length about 575 $m\mu$). Two observers have selected glasses from this series requisite to produce a scale progressing by equal steps from white to yellow. The step size varied from something approaching the least perceptible in one determination to about four times that size in another; it was found that this variation produced no significant change in the shape of the curve connecting purity and saturation. An increase in step size by an additional factor of three was tested for five observers. It was found chiefly to introduce greater individual difference; the average for all observers was not significantly changed. It is concluded that the integral of the sensibility to purity change from zero purity may safely be taken as the saturation of the yellow color evoked under the present observing conditions. The saturation-purity curve resulting from these determinations is compared with that from previous experimental data and with those from two empirical relations. In order to discover whether the discrepancies revealed by this comparison are ascribable to individual difference, groups of glasses representing each of four curves compared were selected and judgments obtained from eight observers with the result that the experimental scale was corroborated. The individual differences shown among these eight observers were too slight to account for the discrepancy. It is concluded that an empirical relation which was previously found fairly well justified by data on other types (wave-length, color-temperature) of chromaticity sensibility, though not perfect, yields a saturation scale for yellow which is close to the true one.

I. INTRODUCTION

SATURATION is defined by Troland[1] as "that attribute of all colors possessing a hue, which determines their degree of difference from a gray of the same brilliance. . . (footnote). It is a subjective attribute of color." This is a purely qualitative definition, but it has been customary to give saturation a quantitative meaning by calling a series of stimuli producing evenly spaced colors of the same hue and brilliance a saturation scale.[2] The saturation of a given color in such a series is measured by the number of steps from the beginning of the series, which is an achromatic color (gray) of the same brilliance. Besides depending upon the accuracy with which the visual intervals between successive members of this series are made identical, the legitimacy of quantifying saturation in this way depends on a number of assumptions, commonly tacit assumptions. The only one which matters here is the assumption

* Publication approved by the Acting Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] L. T. Troland, *Report of the Colorimetry Committee for 1920–21*, J. O. S. A. and R. S. I. **6**, 534–535 (1922).

[2] L. A. Jones and E. M. Lowry, *Retinal Sensibility to Saturation Differences*, J. O. S. A. and R. S. I. **13**, 25–34 (1926).

BOSTON UNIVERSITY
COLLEGE OF LIBERAL ARTS
LIBRARY

that the numbers purporting to be saturation are independent of the size of the visual interval between successive members of the series; this assumption would be proved wrong if it were found that a series made up of, say, every fifth stimulus of the original scale produced colors for which the successive differences no longer were equal.

It is the present purpose (1) to set up a saturation scale for yellow, (2) to test whether the size of the interval affects the scale, (3) to compare the saturation scale with previous results both direct and inferred from observations dealing with other hues, and (4) to discover whether discrepancies in published data may be referred to individual difference.

II. Experimental Method

The stimuli making up the saturation scale for yellow were produced by illuminating Lovibond yellow glasses[3] with light from a milk glass illuminated in turn by light chiefly from the overcast north sky. The dominant wave-length of this series of glasses for Abbot-Priest sun as illuminant and heterogeneous stimulus varies from 573 for the low numbers to 575 $m\mu$ for the high numbers, but there is no noticeable regular variation in hue. For increasing Lovibond yellow number, the colors produced increase in saturation and are of nearly constant hue. There exists, of course, also a decrease in brightness; but the effect of this brightness difference in comparing glasses of different Lovibond number was eliminated by using the Martens photometer.[4]

A single determination of the saturation scale involved the following processes: (1) The Lovibond yellow glass corresponding to one extreme of the saturation scale (either "clear glass" or 20Y) was adjusted so that it was seen in one-half of the 6° circular field of the Martens photometer; call this glass, glass (1). (2) A choice of another glass (2) was made such that the initial saturation step that it produced constituted a convenient standard to be maintained throughout the run. This size of step was varied from one to, perhaps, four "least perceptible differences" under these experimental conditions. (3) After scrutinizing, with brightness equalized, the saturation step so produced (1–2), the observer was required to choose a yellow glass (3) which produced a second step (2–3) of saturation equal to the first step (1–2). (4) The result of the third process was checked by requiring the observer to compare again the saturation step (2–3) with the saturation step (1–2). If the observer still judged these steps to be equal, the glass chosen by the third process was accepted. (5) Then the observer was required to select a fourth glass (4) such that step (2–3) was equal to step (3–4), and so on until the other extreme of the saturation scale was reached.

This method may be called the method of equal sense differences; it differs from the more usual method of successive liminal determinations first, and non-essentially, by reason of the larger steps, and second because the observer is enabled to obtain a constant check on the size of step to be chosen.

Six determinations, each on a different day, were made by one observer (DBJ); and three by another (JOR). Of these nine determinations, five started at the lower, four at the upper end of the scale.

III. Comparison with Previous Results

In comparing these results with previous saturation scales for yellow, it is convenient to specify the stimuli in terms of their colorimetric purities.[5] This convenience arises from the fact that if the other attributes of the visual stimulus, dominant wave-length and brightness, are kept nearly constant and within the usual limits, the changes in colorimetric purity of the stimulus produce colors of nearly constant brilliance and hue, but of widely varying saturation. On this account colorimetric purity may be taken as the

[3] The Tintometer (Ltd), The Colour Laboratories, Salisbury, England; 1912. K. S. Gibson and F. K. Harris, *The Lovibond Color System, I. A Spectrophotometric Analysis of the Lovibond Glasses*, B. S. Sci. Paper No. 547; Feb. 17, 1927.

[4] F. F. Martens, *Über ein neues Polarisationsphotometer*, Phys. Zeits. **1**, 299–303 (1900). D. B. Judd and G. K. Walker, *A Study of 129 Lovibond Red Glasses with Respect to the Reliability of Their Nominal Grades*, Oil and Fat Industries **5**, 17–18 (1928).

[5] I. G. Priest, *Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity, and Brightness*, J. O. S. A. and R. S. I. **8**, 173–200 (1924). *The Computation of Colorimetric Purity*, J. O. S. A. and R. S. I. **9**, 503–520 (1924). D. B. Judd, *The Computation of Colorimetric Purity*, J. O. S. A. and R. S. I. **13**, 133–154 (1926).

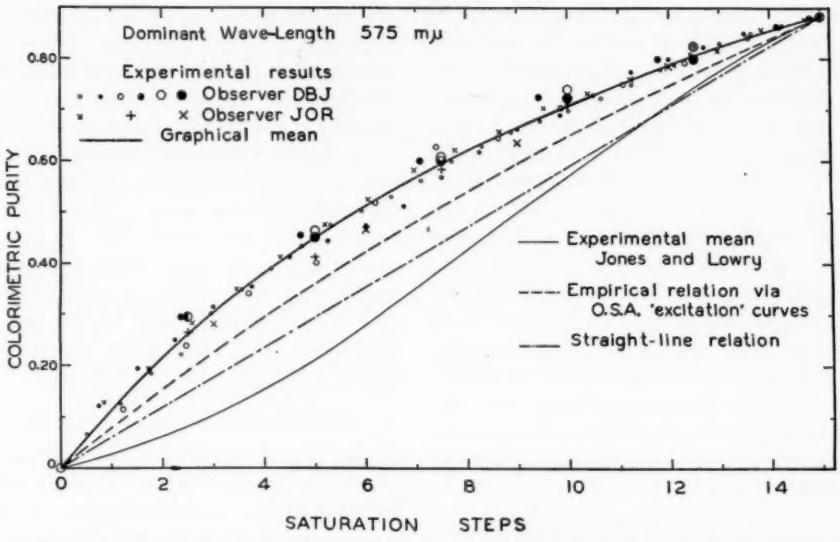


FIG. 1. Colorimetric purity of the stimulus as a function of the saturation of the colors produced. The present experimental results are compared to a previous experimental mean and to two empirical relations.

stimulus analogue of the subjective attribute, saturation. Indeed, so close and striking is the connection between the stimulus entity, colorimetric purity, and the response entity, saturation, that the two terms have frequently been confused.[6]

The purities of the glasses were read from a curve drawn smoothly among points representing the purities of the unit glasses from $N=1$ to $N=20$, also for $N=0.1$ and $N=0.01$. These values were computed from the spectral transmissions of the glasses previously determined by Gibson and Harris.[7] In these computations the O.S.A. "excitation" curves in their extrapolated form[8] were used, and Abbot-Priest sunlight[9] was taken as the heterogeneous stimulus and as the illuminant. Although the light from the overcast north sky passing through the slightly greenish-yellow milk glass which was used is not an accurate duplication of Abbot-Priest sunlight, it is known that the degree of duplication is amply sufficient for this purpose.

Fig. 1 shows the experimental data in terms of colorimetric purity plotted against the number of steps; the total saturation difference between $N=0$ and $N=20$ is taken arbitrarily at fifteen steps; this is a convenient choice because the size of interval is then about equal to one step of the Jones hue scale.[10] It is possible under our experimental conditions to detect more than fifteen differences between $N=0$ and $N=20$; for example, one determination shown in the figure requires twenty-five small circles for its representation. But, since the purpose is merely to determine

[6] D. McL. Purdy, *On the Saturations and Chromatic Thresholds of the Spectral Colours*, Brit. J. Psych. (General Section) **21**, Part 3, 284 (1931). D. B. Judd, *A General Formula for the Computation of Colorimetric Purity*, J. O. S. A. **21**, 741–747 (1931).

[7] See reference 3. Additional data required, and all computations of dominant wave-length and purity, were already available as part of the general standardization of Lovibond glasses.

[8] Spectrophotometry, *Report of the Optical Society of America Progress Committee for 1922–23*, J. O. S. A. and R. S. I. **10**, 230 (1925).

[9] I. G. Priest, *A Precision Method for Producing Artificial Daylight*, Phys. Rev. [2] **11**, 502 (1918). D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, B. S. J. Research **4**, 525 (1930).

[10] L. A. Jones, *The Fundamental Scale of Pure Hue and Retinal Sensibility to Hue Differences*, J. O. S. A. **1**, 63–77 (1917). The Jones "hue scale" also appears in the *Report of the Colorimetry Committee of the O. S. A., 1920–21*, J. O. S. A. and R. S. I. **6**, 545 (1922), under the title, *Spectral Chroma Scale*, which is consistent with the terminology recommended in that report.

the shape of the curve, the particular size of step chosen in this comparison is of no consequence.

It may be seen from Fig. 1 that results by the two observers are in good agreement, and that the variation in size of step by a factor of four does not change the shape of the curve significantly. Later checks by five observers with the same glasses used in a similar, though less direct way has shown that a further increase in size of interval by a factor of three likewise produces no significant effect on the saturation scale; it did show that the use of very large steps, steps 7 or 8 times the size of one step on the Jones hue scale, introduces considerable uncertainty into the determination and increases markedly the individual differences between observers, but the average results of five observers agreed closely with the experimental results indicated in Fig. 1 by the heavy solid curve drawn among the plotted points. A previously reported constant error due to large size of step[11] was proved by this check to be not representative of the average observer. It may be concluded, therefore, that the saturation scale for yellow is independent of the step size used in its determination by an average observer. Hence, saturation of yellow colors may be defined quantitatively as the integral of the purity sensibility from zero up to purity, p. This corroborates for one hue the tacit assumption commonly made for all hues.[12]

There is only one previous determination of a saturation scale for yellow with which to compare our experimental results; this is the determination by Jones and Lowry[2] for dominant wave-length 575 $m\mu$; these results are given in Fig. 1 by the light solid curve.

Sensibility to purity change has been determined for other hues, however, and from these data it has been independently suggested by Munsell[13] and Donath[14] that sensibility to purity change for any given hue is approximately constant independent of purity. If we take, as is usual, the integral of the purity sensibility from zero up to purity, p, as a quantitative expression for saturation, this constancy leads, of course, to a straight-line relation between saturation and purity. This proportionality has doubtless contributed to the confusion, previously mentioned[6] between the two terms. The relation is represented in Fig. 1 by the straight line (dot-dash) connecting the extremes of the other curves.

A second relation has been suggested as a representation of sensibility to purity change, dE/dp, as a function of purity, p:[15]

$$dE/dp = S(3C-1)/K[3S(1-p)+p]^2 \quad (1)$$

where S and C are functions of wave-length simply defined from the O.S.A. excitation curves, and K is a constant depending on the size of the steps in terms of which the color difference, dE, is expressed. (For dE in steps of the size of those in the Jones hue scale, K is approximately 0.01.) This relation has been found to represent Donath's data somewhat more closely than the straight-line relation which he suggested. It is a special case of the general empirical expression which has been found to accord approximately with sensibility data of a number of kinds (wave-length, color-temperature) in addition to sensibility to purity change. According to this expression, the saturation difference between two colors of the same hue is proportional to the distance on the Maxwell triangle between the two points representing the colors. Integration of (1) from zero purity up to purity, p, gives:

$$\int_0^p (dE/dp)dp = p(3C-1)/3K[3S+p(1-3S)]. \quad (2)$$

For dominant wave-length equal to 575 $m\mu$, $C=R=0.51575$,[16] $S=0.48634$; and if we take $K=0.01017$ for the sake of accord with the arbitrarily chosen step size, there results:

$$\text{Number of saturation steps} = 54.7\ p/(4.45-1.40\ p). \quad (3)$$

[11] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. O. S. A. **22**, 99 (1932). The two similar cases mentioned for red may likewise prove to be not representative.

[12] See Introduction.

[13] A. H. Munsell, *On the Relation of the Intensity of Chromatic Stimulus (Physical Saturation) to Chromatic Sensation*, Psych. Bull. **6**, 238–239 (1909).

[14] F. Donath, *Die funktionale Abhängigkeit zwischen Reiz und Empfindung bei der Farbensättigung*, Neue Psych. Stud. **2**, 143–207 (1926).

[15] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. O. S. A. **22**, 91 (1932).

[16] D. B. Judd, *Interpolation of the O. S. A. "Excitation" Data by the Fifth-Difference Osculatory Formula*, J. O. S. A. **21**, 531–540 (1931).

This is the equation of the dashed curve of Fig. 1. It will be noted that, although this equation fails significantly to represent our experimental results, it agrees with them better than the determination of Jones and Lowry made under different experimental conditions; it is also a somewhat better representation of them than the straight-line relation.

IV. Dependence on Individual Observer

It is, perhaps, not surprising that the straight-line relation fails to represent our experimental data; we should regard the straight-line relation at best merely as a first approximation.[17] Likewise is the agreement with the experimental results of the empirical relation via the O.S.A. excitation curves about all that should be expected; other data have shown the general empirical relation, from which Eq. (3) was derived, to be unreliable by as much as a factor of two in other special cases. The failure, however, of our experimental results to agree with the direct determinations of Jones and Lowry deserves consideration; as may be judged from the points plotted in Fig. 1, the discrepancy is three or four times too large to be explained by our experimental error. Since only two observers were used in each case, and since it is conceivable that observers may differ importantly in their judgment of the relative sizes of two saturation differences, the suspicion was justified that individual difference might be the source of the discrepancy. Accordingly, a convenient method of checking the importance of individual difference was devised which also has the advantage of introducing experimental conditions resembling those usual in comparing the colors of material objects.

The saturation scales indicated by the four curves of Fig. 1 were set up by Lovibond yellow glasses, eight glasses, placed side by side, in each scale. The four scales were arranged one above the other and illuminated by a large milk glass screen (brightness approximately 50 millilamberts) which, in turn, was illuminated by light from the overcast north sky. The observer's eyes were separated from the vertical plane of the Lovibond glasses by about one meter so that the row of glasses making up the scale subtended about 10 degrees at the observer's eyes, and any two consecutive glasses formed an approximately square field about 2° across and divided vertically. The surrounding field (brightness about 1 ml) was gray and approximately 40° in extent. A similar arrangement was used in comparing three eleven-glass scales, the scale according to Jones and Lowry being omitted.

This arrangement is advantageous in that it permits the observer to use both eyes in a well-accustomed way; on the other hand, noticeable differences in brilliance, as well as saturation differences, were present and had to be discounted by the observers.

The observer was asked to point out the scales in order of their approximation to perfection. The perfect, or ideal, scale was defined as one in which the saturation steps are equal. The observer was also asked to point out the specific faults in his second, third, and fourth choices which resulted in ranking them lower than first. The responses of eight observers were noted both for the eleven-glass and for the eight-glass scales, two of the most experienced observers (IGP, KSG) repeating their criticisms of the eight-glass scales on another day, making 18 separate judgments for three of the scales, and 10 for the fourth. The observers were ignorant of the identities of the scales. A summary of these responses is given in Table I.

An inspection of the table shows that the experimental scale obtained by the method of

Table I. *Distribution of the judgments among the four saturation scales.*

Judgments	Judd and Riley	Eq. (3)	Straight-line	Jones and Lowry
Best scale	15	3	0	0
Second best	3	14	1	0
Third best	0	1	15	2
Fourth best	0	0	2	8
Steps too large at low saturation	4	0	0	0
Equal steps	9	2	0	0
Steps too small at low saturation	5	16	18	10

[17] For some hues, blue and violet, the straight-line relation between saturation and colorimetric purity would seem to be not even approximately correct. Observation by one observer (DBJ) for dominant wave-length 452 $m\mu$ indicates this relation to be in error by a factor of 40 (J. O. S. A. **22**, 94 (1932)).

equal sense differences was preferred; it may be noted, also, that the scale set up according to Eq. (3) is satisfactory to some observers. It may be seen, however, that individual difference is not a source of sufficient uncertainty to account for the discrepancy between our experimental results and those of Jones and Lowry. This discrepancy remains unexplained; it might be due to (1) instrumental error, (2) difference in psychophysical method, or (3) difference in experimental conditions.

The approximate corroboration of the empirical relation based on distances on the Maxwell triangle confirms the use of trilinear coordinates to facilitate the writing of uniform color tolerances.

A Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight*

IRWIN G. PRIEST, *Bureau of Standards, Washington, D. C.*

(Received December 9, 1932)

THE PROPOSAL IN BRIEF

IT is proposed that reciprocal temperature (instead of temperature itself) be adopted as a conventional parameter for specifying the chromaticity[1] of incandescent illuminants and various phases of daylight. It is recommended that the micro-reciprocal-degree absolute centigrade be adopted as the most convenient unit for expressing reciprocal temperatures. The following reasons are advanced for proposing these conventions:

(1) A given small interval in reciprocal temperature is (approximately) equally perceptible as a difference in chromaticity regardless of the color temperature. In other words, a "just-perceptible difference" (or other small difference of given perceptibility) in reciprocal temperature is an (approximately) constant quantity independent of the value of color temperature. Specification directly in terms of reciprocal temperature is therefore much more significant than in terms of temperature. For example, if we specify the chromaticities of sources in terms of color temperature, we find that a difference of 100° at 20,000°K is no more easily perceptible as a difference in chromaticity than is 1° at 2000°K.

(2) A difference of one micro-reciprocal-degree is fairly representative of the doubtfully perceptible difference in chromaticity under the most favorable conditions of observation. The unit is thus of convenient size and its significance is readily remembered. The numerical values of the proposed scale, corresponding to color temperatures 2000°K to 20,000°K, will run from 500.0 to 50.0. Thus, a scale of integers from 50.0 to 500.0 will suffice for "rough-and-ready" specification of the chromaticities of all common natural and artificial illuminants of the "black-body" type. The giving of one decimal place will constitute a rather precise specification, and there will never be occasion to give more than two decimal places.

(3) Many computations are greatly facilitated by dealing with reciprocal temperature instead of temperature. Even when we are obliged to specify temperatures on account of conventional practice, it is necessary to take the reciprocals of these temperatures in computation. Since these reciprocals serve the purpose of specification just as well as the temperatures themselves, it appears that we might as well neglect consideration of the latter and thereby effect a considerable saving in labor as well as a gain in convenience.[2] A great deal of time may be saved and thought may be clarified and expedited by substituting reciprocal temperature for temperature; but nothing is gained and considerable confusion may be introduced by substituting temperature for reciprocal temperature.[3]

(4) The substitution of reciprocal temperature for temperature is simple and the relation is easily remembered, whereas the translation from temperature to spectral centroid (as previously proposed tentatively by the present writer) is a

* Publication approved by the Acting Director of the Bureau of Standards of the U. S. Department of Commerce. The manuscript of this paper was completed by Mr. Priest just prior to his death on July 19, 1932.

[1] For meaning of the term "chromaticity" see *O. S. A. Colorimetry Report, 1920–21*, J. O. S. A. **6**, 535 (1922), footnote 8.

[2] These considerations apply to strictly pyrometric as well as colorimetric computations.

[3] Consider, for example, the formulas of Wien and Planck for spectral distribution of energy; and see especially the following papers for illustration of the convenience of reciprocal temperature in computation and specification: Langmuir and Orange, Trans. A. I. E. E. **32**, 1946 (1913); Ch. Fabry, Trans. I. E. S. **8**, 317–318 (1913); Foote, Mohler and Fairchild, J. Wash. Acad. Sci. **7**, 545–549 (1917); H. P. Gage, Trans. I. E. S. **16**, 428–429 (1921). Note especially the convenience and simplicity of reciprocal temperature in expressing the additive value of a blue filter in changing the apparent reciprocal color temperature of a source viewed through this filter.

tedious matter more or less likely to introduce confusion and misunderstanding.

Notes Concerning the Circumstances of this Proposal and the Events Leading to It

For a number of years past it has been found convenient to specify the chromaticity of various illuminants by specifying the so-called "color temperature."[4] While this parameter has, in many cases, some interest as *temperature* it is to be noted at once that, in specifying the chromaticity of illuminants, we are not interested directly in "temperature." We are merely using the scale of temperature as a scale for arranging the chromaticities of the several illuminants in a serial order.[5]

[4] The practical use of the color of an incandescent solid as an index of other properties dependent fundamentally upon temperature, even though the observer may have had no concept of temperature, is an exceedingly ancient practice, the origin of which is lost in the mists of antiquity. It was a technique doubtless used by ancient prehistoric smiths. In recent times, within the past century, this practice of estimating temperature by means of observations on the color of a furnace or an incandescent body has been developed and used industrially to a considerable extent but largely by tradition and "rule of thumb" without definite scientific formulation. In connection with the development of the incandescent filament lamps, the concept of color temperature finally came to be formulated much more definitely. The formal definition of the term color temperature, the relation of color temperature to the several "brightness temperatures" and the relation of color temperature and brightness temperature to "true temperature" were discussed comprehensively by Hyde, Cady, and Forsythe in their paper *Color Temperature Scales for Tungsten and Carbon*, Phys. Rev. [2] **10**, 395–411 (1917). Prior to this, Patterson and Dudding in their paper on *The Estimation of High Temperatures by the Method of Color Identity* (Proc. Phys. Soc. (London) **27**, 230–262 (1915)) had made extensive use of the concept of color temperature which they called "the color identity temperature." Hyde and his several associates had used the concept of color temperature in a quite definite manner in several papers earlier than this, although the *term* color temperature was apparently not used in these earlier papers. See particularly the following: Hyde, *New Photometric Methods of Studying the Radiating Properties of Various Substances*, Phys. Rev. **27**, 521–522 (1908); Hyde, Cady, and Middlekauff, *The Selective Emission of Incandescent Lamps as Determined by New Photometric Methods*, Trans. I. E. S. **4**, 334–354 (1909); Hyde, *The Physical Production of Light*, J. Frank. Inst. **169**, 439–466 and **170**, 26–45 (1910); Hyde, *The Synthetic Development of Radiation Laws for Metals*, Astrophys. J. **36**, 89–132 (1912); Langmuir, *Tungsten Lamps of High Efficiency—I*, Trans. A. I. E. E. **32**, 1913–1933 (1913); Langmuir and Orange, *Tungsten Lamps of High Efficiency—II*, Trans. A. I. E. E. **32**, 1935–1946 (1913).

The idea of grading illuminants in terms of color temperature was further developed and illustrated by Hyde and Forsythe in their paper *Color Temperature and Brightness of Various Illuminants*, Trans. I. E. S. **16**, 419–430 (1921). The definition of the term color temperature was given further consideration in the discussion which followed this paper (pp. 429–430). The following are subsequent papers bearing on this same subject by Hyde's associates at the Nela Research Laboratory: Forsythe, *Accuracy in Color Matching of Incandescent Light Sources*, J. O. S. A. **6**, 476–482 (1922); Forsythe, *The Temperature and Brightness of Tungsten Lamps*, Gen. Elec. Rev. **26**, 830–834 (1923); Forsythe, *Color Match and Spectral Distribution*, J. O. S. A. **7**, 1115–1121 (1923); Forsythe and Worthing, *The Properties of Tungsten and the Characteristics of Tungsten Lamps*, Astrophys. J. **61**, 146–185 (1925); Forsythe, *Temperature Radiation*, J. O. S. A. **16**, 307–328 (1928).

[5] It should be noted that in giving color temperatures for this purpose we should adhere strictly to the following definition: "*The color temperature of a radiator (source, lamp) is the (absolute) temperature at which a complete radiator must be operated in order to emit energy competent to evoke a color of the same chromaticity as the color evoked by the radiant energy from the source in question.*" Cf. Priest, J. O. S. A. and R. S. I. **7**, 1180 (1923). For certain other purposes a slightly different definition of color temperature is sometimes adopted either explicitly or tacitly. Such a definition is: "that temperature of a black body at which the relative emission intensities in some chosen two wave-lengths are the same as those of the radiating metal under investigation." (Phys. Rev. [2] **10**, 396 (1917). The adoption of this definition would not be appropriate to our purpose of *color-grading* sources and in some cases the choice of the definition might make a significant practical difference in the statement of color temperatures characteristic of certain sources. It is for this reason that certain so-called physical methods for the determination of color temperature (Campbell and Gardiner, J. Sci. Inst. **2**, 177–187 (1925); Winch, J. Sci. Inst. **6**, 374–379 (1929); Sharp, J. O. S. A. **20**, 62–70 (1930)) are not entirely adequate for the *color-grading* of sources. They would be fully adequate only in case it were known that the chromaticity of the source being investigated would match the chromaticity of the standard (the complete radiator) when the source in question had the same ratio of intensities as the standard for two specified wave-lengths (or specified spectral regions).

The term "color temperature" is not an entirely happy one from the point of view of colorimetrics. It was once remarked by Dr. H. E. Ives that, where our interest is in specifying the quality of color (chromaticity) of the source rather than the temperature of the source, it would be logical to speak of temperature color rather than color temperature. Perhaps an improvement in terminology might be made in connection with the present proposal by giving the name "thermal chromaticity" or the name

Initially, the usage of color temperature had to do almost exclusively with the common artificial illuminants ranging in color temperature from about 1800° absolute to about 3000° absolute. It was later found useful and convenient to grade the various phases of daylight on the scale of color temperature and this necessitated extending the scale to over 20,000° absolute.[6]

It appeared that the chromaticities of all common incandescent illuminants and the more important varieties of daylight could be specified approximately by stating appropriate color temperatures. In astrophysics, it has also become a more or less common practice to give the color temperatures of the stars.[7] It also appeared, however, that the perceptibility of a given interval in degrees depended in a marked manner upon the temperature. For example, a difference of one degree at 2000°K corresponds to a difference in chromaticity no less than that corresponding to a difference of 100° at 20,000°K. That is to say, a difference of one degree in temperature at 2000° is as readily detectable chromatically as a difference of 100° at 20,000°K. Such a scale is manifestly inconvenient. It has always been evident that the specification of the chromaticity of sources by color temperatures higher than about 4000° was inconvenient and misleading and, to a certain extent, ludicrous. Nearly ten years ago, I suggested that this inconvenience of the color temperature scale might be obviated by adopting the convention of specifying the "spectral centroid" instead of the color temperature of a source.[8] I had merely observed empirically that equal increments in the spectral centroid were equally perceptible for all color temperatures. Although this is a noteworthy advantage of the spectral centroid scale, it must be admitted that it is offset by notable disadvantages. Among these are the following: (1) It is a laborious and tedious task to compute the spectral centroid as a function of temperature. (2) To render the scale significant one must always use the same visibility function in computing the spectral centroid. The visibility function which I used for this purpose has since been superseded by that adopted by the International Commission on Illumination. (3) There is no simple, easily remembered relation between temperature and spectral centroid.

This subject has now acquired new interest from recent measurements made by my associate, Dr. Judd,[9] which show that the empiric relation which I pointed out is valid over a greater range of temperature than I had tested it, and is a more accurate representation of experimental fact than we had reason to believe previously. The essence of these new observations, insofar as they interest us in this connection, may be summarized as follows: (1) The doubtfully perceptible difference was determined by each of six observers for the following values of reciprocal temperature $(1/\theta)$: 567, 452, 348, 198, 91 micro-reciprocal-degrees. (2) Expressed in terms of spectral centroid, the average doubtfully perceptible difference for all six observers (*under the particular experimental conditions of these observations*)[10] at all five values of $1/\theta$ was $\Delta\lambda_c = 0.385$ $m\mu$, the maximum and minimum values for individual observers (averaged with regard to $1/\theta$) being respectively 0.51 and 0.31 $m\mu$, these data indicating that the variation with $1/\theta$ is either nonexistent or less than 20 percent.

Soon after I had pointed out that a given difference in spectral centroid was approximately equally perceptible for all temperatures over a wide range, Gage pointed out that there must be

"radiant chromaticity" to the reciprocal of color temperature as defined here.

[6] Priest, *The Complete Scale of Color Temperature . . .*, Phys. Rev. [2] **20**, 93–94 (1922); Priest, *Colorimetry and Photometry of Daylight and Incandescent Illuminants . . .*, J. O. S. A. **7**, 1175–1209 (1923), particularly pp. 1190–1192, and Fig. 2 on p. 1184. See also Priest, *A Method of Obtaining Radiant Energy Having the Visible Spectral Distribution of a Complete Radiator at Very High Temperatures*, J. O. S. A. **5**, 178–183 (1921).

[7] See, for example, Smithsonian Physical Tables, 7th Ed., 3rd reprint, p. 411 (1927).

[8] J. O. S. A. **7**, 1190–1192 (1923), and Fig. 2 on p. 1184.

[9] Judd, J. O. S. A. **23**, 7 (1933).

[10] Note carefully, however, that the conditions of these experiments were not such as to facilitate the finest discrimination, the purpose of these experiments being merely to obtain relative values of *equally perceptible differences* for different values of $1/\theta$, not to determine the very least values of this difference under the most favorable conditions for making the difference small. Under conditions of observation highly favorable to precise observation, the doubtfully perceptible difference is found to be approximately one-fifth of the values found in these experiments. (See Judd, Bur. Standards J. Research **5**, 1161–1177 (1930).)

a linear relation between spectral centroid and the reciprocal of temperature.[11] This empiric relation is illustrated in Fig. 1. From this relation pointed out by Gage it follows that, insofar as a given difference in spectral centroid is equally perceptible at all temperatures, a given difference in reciprocal temperature is also equally perceptible at all temperatures.

Independently, Davis has also pointed out that, insofar as a given difference in spectral centroid is equally perceptible at all temperatures, a small difference in temperature of constant perceptibility must vary directly as the square of the temperature.[12] It thus appears that the following two relations may be derived from the spectral centroid relation: (1) Temperature differences equally perceptible by difference in chromaticity vary directly as the square of the absolute temperature. (2) The difference of given perceptibility in reciprocal temperature is a constant for all temperatures. It is evident that these two statements are merely two modes of expressing the same relation.[13] We shall find the second mode of expression more convenient and more useful.

It is impossible to decide with certainty, even from Dr. Judd's rather extensive experimental data, whether or not the doubtfully perceptible difference in spectral centroid is more nearly constant than the doubtfully perceptible difference in reciprocal temperature. Both relations are merely empiric. The latter is much more convenient of application. It is therefore recommended that reciprocal temperature be adopted as a convenient parameter for grading the chromaticity of light sources.[14]

While the chromaticity of a great many practical and common illuminants may be approximately specified by giving the temperature at

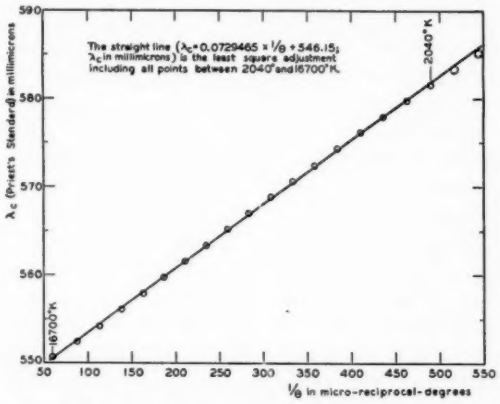


FIG. 1. Relation between spectral centroid and reciprocal temperature. According to the slope of this line, 1.0 $m\mu$ corresponds to 13.709 micro-reciprocal-degrees.

which the complete radiator would approximately match them, it is to be noted that in practically all cases there would be no temperature at which the theoretically complete radiator would *exactly* match in chromaticity one of these practical sources. These small departures may be specified in various ways. For example, one may specify the filter which would have to be combined with the complete radiator at a given temperature in order that the chromaticity of this source-and-filter combination should be accurately the same as that of the source in question. Knowing the spectral transmission of this (very slightly selective) filter, the trilinear coordinates of the source

[11] Letter, H. P. Gage, to K. S. Gibson, December 9, 1925. Dr. Gage had noticed that a linear relation existed between the thickness of a plate of "Daylite" glass and the difference between the reciprocal of the color temperature of a source and the reciprocal of the apparent color temperature of the same source viewed through the plate of Daylite glass. Dr. Gibson had pointed out that a linear relation existed between the thickness of a plate of Daylite glass and the spectral centroid of a source as seen through the plate of Daylite glass. (J. O. S. A. 11, 473–476 (1925).) From these two statements Dr. Gage drew the conclusion that a linear relation must exist between the spectral centroid and the reciprocal of the color temperature.

[12] R. Davis, B. S. Res. Pap. No. 365, 672 (1931). There is a typographical error in the ninth line from the bottom of this page, . . . θ should be small δ which is the temperature difference (measured in degrees) which is equally perceptible at all values of temperature over a wide range. The essence of the relation which Davis points out is contained in his equation, $\delta = K\theta^2$ where K is a constant.

[13] Note that the rate of change of temperature with respect to reciprocal temperature is minus the square of temperature. In symbols $d\theta/d(1/\theta) = -\theta^2$. With sign reversed, the proportionality constant in the equation stating Davis' rule (see reference 12) is equal to the increment in reciprocal temperature corresponding to the doubtfully perceptible difference in temperature.

[14] It should be noted, however, that the relation between λ_c and $1/\theta$ is not *exactly* linear and there are circumstances in which one should use a more accurate empiric relation for obtaining values of $1/\theta$ corresponding to given values of λ_c, and conversely.

in question may be computed on the basis of given standard coordinates for the complete radiator at the temperature specified. Davis has indicated the theory of specifying the chromaticity of any source by specifying the temperature of a complete radiator and the wave-length of homogeneous energy (Davis' "conjunctive wave-length") which would have to be added to the energy from the complete radiator at this temperature in order to match the source in question.[15] Hyde and Forsythe[16] had previously suggested the experimental counterpart of the idea which was used by Davis in connection with computation of color temperatures from data on spectral distribution.

Precision and Accuracy of Color-Matching in Terms of Reciprocal Temperature

The probable error of a single observation of color-match varies from about 1.0 micro-reciprocal-degree for small fields (3°) and slightly unfavorable conditions of observation to about 0.28 micro-reciprocal-degree for large fields (20°) and very favorable conditions of observation.[17] If we take the value of the "doubtful difference" as 3.3 times the probable error of a single observation,[18] we find that the doubtful difference under the most favorable conditions of observation is about one micro-reciprocal-degree. For further illustration, we may give data computed from Tables I and II in B. S. Sci. Pap. No. 443.[19] These data illustrate the precision and the agreement obtained by different observers in color-matching lamps by means of the Martens photometer with its 6° field, the brightness match being attained by adjustment of the photometer while the chromaticity match is attained by adjustment of a resistance in series with a lamp. Four different lamps were each matched with a standard lamp by the substitution method by each of four observers. Two sets of ten observations each were made by each observer for each of the four lamps. From all of these data, the average probable error of a single observation of color-match is found to be 0.69 micro-reciprocal-degree, ranging from 0.57 to 0.78 micro-reciprocal-degree for the different observers.[20] The average departure of an individual observer's final value from the mean of the four observers' values for a single lamp is 0.65 micro-reciprocal-degree. The greatest departure of an individual observer's value from the mean of the four observers' values for a given lamp (considering all four observers' data for all four lamps)[21] is 1.25 micro-reciprocal-degrees (10.2°).

From the data just given it is evident that the statement of reciprocal temperature to the nearest unit in micro-reciprocal-degrees is sufficient for specifying the chromaticity of a source in all cases except where the highest possible precision is desired and is justified by taking the mean of a large number of observations, in which case reciprocal temperatures may be given to 0.1 micro-reciprocal-degree. Rarely, if ever, will one specify reciprocal temperature to 0.01 micro-reciprocal-degree although this degree of formal precision may sometimes be observed in specifying fundamental standards even if not entirely justifiable on the basis of experimental accuracy.

[15] B. S. Res. Pap. No. 365, 677–678 (1931).

[16] Forsythe, Trans. I. E. S. **16**, 423–424 (1921).

[17] Computed from data given by Judd in terms of spectral centroid (λ_c), B. S. Res. Pap. No. 252, Fig. 5, 1175 (1930).

[18] B. S. Res. Pap. No. 252, 1170–1171 (1930).

[19] Priest, *Measurement of the Color Temperature of the More Efficient Artificial Light Sources by the Method of Rotatory Dispersion*, B. S. Sci. Pap. No. 443, July (1922).

[20] B. S. Sci. Pap. No. 443, Table I, p. 223. Considering all of these data, it is also found that the ratio of the maximum deviation of a single observation from the mean of ten to the probable error of a single observation varies, for the different observers, from 4.1 to 5.0.

[21] B. S. Sci. Pap. 443, Table II, p. 224.

Color Filters for Altering Color Temperature. Pyrometer Absorption and Daylite Glasses

H. P. GAGE, *Optical Laboratory, Corning Glass Works, Corning, N. Y.*

(Received July 30, 1932)

The analysis of the Wien equation shows that a color filter in which the density, δ($\delta = \log_{10}$ transmission) follows the equation $\delta = (6223/\lambda)(1/\theta_1 - 1/\theta_2)$ will reduce both the brightness and the color temperature of a complete radiator at temperature θ_1 to that of a complete radiator at a lower temperature θ_2. Such a color filter is useful as an absorbing screen in an optical pyrometer. If a color filter has a spectral transmission such that the logarithm of transmittance plotted against $1/\lambda$ yields a straight line cutting the $1/\lambda$ axis in a point corresponding to approximately $\lambda_1 = 0.40\mu$, such a filter will serve to increase the color temperature of a source although reducing its brightness. The spectral formula of such a filter is $\beta t = 0.4343 C_2(1/\lambda_1 - 1/\lambda)(1/\theta_2 - 1/\theta_1)$, θ_1 being the color temperature of the light source and θ_2 that of the transmitted light expressed in accordance with the Wien equation. With a given filter the reciprocal color temperature is raised by the same amount irrespective of the color temperature of the source and the change in reciprocal color temperature is proportional to the thickness of the glass color filter used. If two points 2 and 3 on the spectrophotometric curve of such a Daylite glass are given, then $1/\theta_1 - 1/\theta_2 = \alpha t = 0.0001607(\beta_2 t - \beta_3 t)/(1/\lambda_2 - 1/\lambda_3)$. With αt to represent the change in reciprocal temperature for the given glass of thickness t, the change per unit thickness is α. Artificial light sources may be described as having a spectral energy distribution in accordance with Wien's equation within the limits of the visible spectrum. Various phases of daylight produce a visual sensation equivalent to the energy distribution as given by Planck's equation or by a suitable but different choice of θ the same distribution may also be represented by Wien's equation within the narrow limits of the visible spectrum, 0.40μ to 0.70μ. A table is included giving the color temperature of some of the artificial light sources and their reciprocals, also the equivalent temperature with its reciprocal for various phases of daylight expressed both according to the Planck and according to the Wien equation.

IN certain previous publications on the subject of absorbing filters for producing artificial daylight,[1, 2, 3, 4] the criterion by which the necessary transmission characteristics of the glass were judged was that starting with the spectral energy distribution of a given light source and the spectral energy distribution of the type of daylight which it was hoped to duplicate, the ratio of the two energy values for each wavelength was the desired transmission for the glass. As illustrated, the energy distribution of the transmitted light was calculated from the energy distribution of the source and the measured transmission curve for a given sample of glass. If the resulting curve agreed fairly well with that of some actual or assumed distribution curve for one of the phases of daylight it was concluded that the light obtainable with the source and filter combination would answer as a substitute for natural daylight. In each case the source and result were considered individually. A possible shortening of the necessary calculations was indicated by Fabry in 1913[5] in which it was shown that with light sources whose energy distribution is equal or closely similar to that of a complete radiator (black body) and therefore expressible by Planck's or Wien's equation there is a generalized relation between the color temperature of the source and that of the transmitted light provided the transmission characteristics of the filter satisfy certain conditions. Special blue glasses were employed by Langmuir and Orange[6] to increase the color temperature of the light from a standard tungsten vacuum lamp to that of a

[1] Herbert E. Ives, *Color Measurements of Illuminants.—A Résumé*, Trans. I. E. S., pp. 189–207, April (1910).

[2] M. Luckiesh and F. E. Cady, *Artificial Daylight—Its Production and Use*, Trans. I. E. S. **9**, 839 (1914).

[3] H. P. Gage, *Artificial Daylight—Discussion*, Trans. I. E. S. **9**, 864 (1914).

[4] Edward J. Brady, *The Development of Daylight Glass*, Trans. I. E. S. **9**, 937 (1914).

[5] C. Fabry, *A Practical Solution of the Problem of Heterochromatic Photometry*, Trans. I. E. S. **8**, 302 (1913).

[6] Langmuir and Orange, *Tungsten Lamps*, Trans. A. I. E. E. **32**, 1946 (1913).

gas-filled lamp at a higher temperature. They used the relation that the difference in reciprocal temperature caused by a special blue glass is a constant and if several such screens are used the the resultant change in reciprocal color temperature is the sum of the reciprocal color temperature changes produced by each screen separately.

Symbols

The symbols used in the following discussion of color filters are:

E_1. The radiant power for a given wave-length incident on the first surface of a color filter.

P_1. The radiant power transmitted by the first surface of the filter is E_1 less the loss by reflection at the first surface.

P_2. The radiant power incident upon the second surface which differs from P_1 because of the absorption of the substance of the filter.

E_2. The radiant power transmitted by the second surface is equal to P_2 less the reflection loss at the second surface.

$T=E_2/E_1$ is called the transmission of the filter.

P_2/P_1 is called the transmittance of the filter and is its transmission corrected for reflection losses.

$\delta=\log_{10}(E_2/E_1)$, density, is the logarithm to the base 10 of the transmission.

t. Thickness of the filter.

$\beta t=\log_{10}(P_2/P_1)$. Logarithm to the base 10 of the transmittance of a filter of thickness t.

β. Transmissive index, logarithm to the base 10 of the transmittance of one millimeter of the filter glass. As the transmission and transmittance of a filter are always less than unity the logarithms δ, βt and β are always negative. Allowing for a reflection factor of 4 percent at each surface, the transmission T is roughly 92 percent of the transmittance. The difference expressed logarithmically is $\delta=\beta t-0.0362$.

λ. The given wave-length expressed in microns.

θ. The absolute or Kelvin temperature of the radiating body.

e. The base of the Naperian logarithms.

$E(\lambda\theta)$. In the Wien and Planck equations the radiant power within equal wave-length intervals $\Delta\lambda$ of average wave-length λ when dealing with a complete radiator at temperature θ.

C_1 and C_2 are constants whose values are subject to experimental determination. For a complete radiator the value of C_1 may be taken as 3.703×10^{-5} ergs cm^{-2} sec.$^{-1}$, i.e., 3.703×10^{-6} microwatts per square centimeter.[7] C_2 has been assigned different values from time to time. Of the more recent values, 14,320 micron degrees is the value adopted in the International temperature scale, 14,330 that adopted in the *International Critical Tables* and 14,350 that used in many computations, such for example as those of Frehafer and Snow[8] and the tables found in the Davis-Gibson paper No. 114[9] which were taken from the Frehafer and Snow data.

$\log_{10} e=0.4343$.

	C_2	$C_2\times0.4343$	$1/(C_2\times0.4343)$
International temperature scale latest value[10]	14,320	6219	0.0001608
Value generally used I.C.T.[7, 11]	14,330	6223	0.0001607
Used in Davis-Gibson paper No. 114[8, 9]	14,350	6232	0.0001604

Absorption Screens Used in Optical Pyrometer

In the optical pyrometer of the disappearing filament type an image of the heated object which is generally of the complete radiator type is formed upon the filament of an electric lamp. This filament is heated electrically until it attains such a temperature that it disappears against the illuminated background produced by the object. This occurs when there is an equality of both chromaticity and brightness. If the filament were a perfect black body (complete radiator) as well as the object, and no absorption or reflection took place between it and the filament this would occur when both are at the same temperature. If, however, this temperature is so high that the life of the lamp filament is reduced or

---

[7] F. E. Fowle, Int. Crit. Tab. V, 238.

[8] M. K. Frehafer and C. L. Snow, *Tables and Graphs for Facilitating the Computation of Spectral Energy by Planck's Formula*, Bur. Stand. Misc. Pub. No. 56 (1925).

[9] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Bur. Stand. Misc. Pub. No. 114.

[10] Wm. F. Roeser, F. R. Caldwell and H. T. Wensel, *The Freezing Point of Platinum*, Bur. Stand. Res. Pap. 326, vol. 6, p. 1119.

[11] W. E. Forsythe, *Intercomparison of High Temperature Scales*, Phys. Rev. **38**, 1247–1253 (1931).

frequent recalibrations would be necessary it may be desirable to insert an absorbing screen between the source and the filament. If the previous assumption is maintained that both the source and the filament are complete radiators and it is further desired not only to reduce the brightness of the background but to cause this background to have the same spectral energy distribution as that of the filament, certain relations between the transmission and wave-length of the absorbing filter are required. Such relations were pointed out by Foote, Mohler and Fairchild.[12] "If the transmission of the absorption glass has the form $T'=e^{\alpha C_2/\lambda}$, we have $1/\theta-1/S=A=\alpha=$a constant independent of temperature." With the usual type of optical pyrometers both filament and background are observed through a red glass. The absorbing screen therefore need follow this relation only between the limits of the transmission of the glass and the sensitivity of the eye, or roughly between 0.61μ and 0.71μ.

Another method of deriving this same relation follows, by using as a basis the Wien equation for the distribution of radiant power throughout the spectrum. While this is not so rigorous as the modified formula of Planck, within the visible range, i.e., from 0.4μ to 0.7μ, the difference between the two is negligible as will be discussed later.

The Wien equation in its usual form is

$$E(\lambda\theta)=C_1\lambda^{-5}e^{-C_2/\lambda\theta}. \quad (1)$$

Expressing this equation in the logarithmic form for ease of calculation:

$$\log_{10} E(\lambda\theta) = \log_{10} C_1+5\log_{10}(1/\lambda)-0.4343C_2/\lambda\theta. \quad (2)$$

The relation between the radiant power for any given wave-length λ and any two temperatures θ_1 and θ_2 is obtained by subtraction. Inasmuch as it is required that the radiant power distribution of the source observed at temperature θ_1, shall be converted to the distribution of the filament at temperature θ_2, the ratio of the radiant powers must express the transmission of the filter E_2/E_1 whose logarithm is δ. We obtain Eq. (3).

$$\delta=\log_{10}(E_2/E_1)=\log_{10}E_2-\log_{10}E_1=-0.4343C_2/\lambda\theta_2+0.4343C_2/\lambda\theta_1=(0.4343C_2/\lambda)(1/\theta_1-1/\theta_2). \quad (3)$$

This equation may be expressed in the form of a curve plotted with δ against $1/\lambda$ and is a straight line passing through the origin, i.e., δ and $1/\lambda$ both equal zero. The meaning of this is that at wave-length infinity there would be perfect transmission.

It is thus evident that the necessary transmission of the pyrometer absorption glass for any given wave-length λ is

$$T_\lambda=E(\lambda\theta_2)/E(\lambda\theta_1), \quad (4)$$

θ_1 being the higher temperature.

Substituting in Eq. (3) the numerical value corresponding to $C_2=14{,}330$, we get

$$\delta=(6223/\lambda)(1/\theta_1-1/\theta_2). \quad (5)$$

As θ_1 is greater than θ_2, $(1/\theta_1-1/\theta_2)$ is negative as required.

Illustration of actual glasses. In 1918 a pair of glasses, one of which (No. 1 in Fig. 1) was a blue green having decreasing transmission towards the longer wave-lengths, the other (No. 2 in Fig. 1) had increasing transmission towards the longer wave-lengths but to a greater extent than was desired, were combined in such proportions that the combination (1+2) had greater transmission towards the longer wave-lengths to approximately the extent desired—between the limits 0.60μ to the end of the sensitivity of the eye at about 0.71μ. The use of two separate glasses being undesirable, an attempt was made to combine two coloring agents in the same glass in such proportions as to secure the desired result. The success of such a glass made in May, 1932 is shown by curve 3, Fig. 1. The use of this glass as an absorption glass in an optical pyrometer is reported[13] "very satisfactory. The difficulty

[12] Paul D. Foote, F. L. Mohler and C. O. Fairchild, *Pyrometry—The Proper Type of Absorption Glass for an Optical Pyrometer*, J. Wash. Acad. Sci. **VII**, 545–549 (1917).

[13] P. H. Dike, Leeds and Northrup Company letter, July 12, 1932, to H. P. Gage.

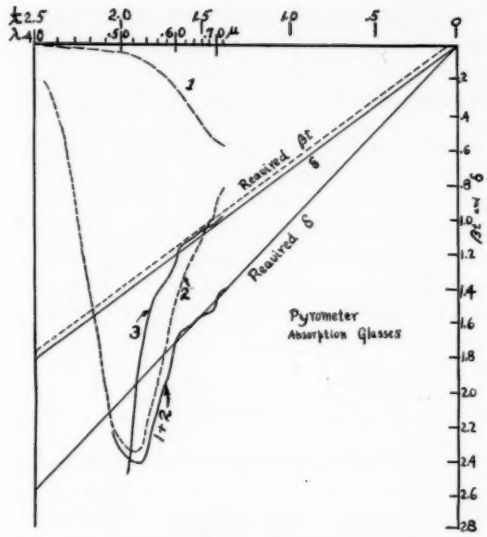


FIG. 1. Pyrometer absorption glasses. Curve 1, blue green glass; curve 2, amethyst glass; curve 1+2, blue green and amethyst together—the ordinates βt or δ are additive when expressed logarithmically; curve 3, pyrometer brown. Beyond wave-length 0.60μ to the limit of the visible spectrum about 0.70μ in the region transmitted by red glass these curves approximate straight lines passing through the origin. The curve 1+2 approximates the line $\delta = 1.03 \times (1/\lambda)$ corresponding to a temperature difference of 166 micro-reciprocal degrees and curve 3 approximates $\delta = 0.728 \times (1/\lambda)$, temperature difference 117 micro-reciprocal degrees.

due to varying shades of red glass filter practically disappears when this glass is used."

DAYLITE GLASS

Daylite glass may be defined as a blue glass which causes the transmitted light to have a higher color temperature than the source without introducing serious spectral irregularities. When such a glass is used it is evident that the value of C_1 in the equation cannot be used because as the color temperature goes up by the use of a thicker glass the brightness must go down. Let us assume that the brightness of two surfaces are nonselectively varied, as, for example, by changing on a photometer bench the respective lamp distances. For any given wave-length it is possible to adjust the brightness of the surfaces to equality but, in general, equality will not then exist for any other wave-length. This may be expressed in Eqs. (1) and (2) for each of the sources by using a value D instead of C_1 to indicate that the relative distribution of radiant power throughout the spectrum is the same but that its intensity is nonselectively reduced. These values of D can also be made to include the constant nonselective reflection factor of the absorbing glass so that instead of using the transmission of the glass it is possible to use the logarithm of the transmittance βt. If the two values of D_1 and D_2 are so chosen that the point of equality of the two curves is at some short wave-length portion of the spectrum in the neighborhood of 0.4μ, this wave-length of equality is designated as λ_1. Substituting D_1 and D_2 for C_1 in Eq. (2) to indicate the reduced values of intensity, we get by subtraction

$$\log_{10} E(\lambda\theta_2) - \log_{10} E(\lambda\theta_1) = \log_{10} D_2 - \log_{10} D_1 - (0.4343 C_2/\lambda)(1/\theta_2 - 1/\theta_1). \quad (6)$$

The values of D_1 and D_2 are so chosen that the intensities for wave-length λ_1 are the same, i.e., $E(\lambda_1\theta_1) = E(\lambda_1\theta_2)$. The first member of Eq. (6) becomes zero when $\lambda = \lambda_1$ and

$$\log_{10} D_2 - \log_{10} D_1 = (0.4343 C_2/\lambda_1)(1/\theta_2 - 1/\theta_1). \quad (7)$$

Substituting this value in Eq. (6) and factoring we get

$$\begin{aligned} \beta t &= \log_{10} (E(\lambda\theta_2)/E(\lambda\theta_1)) \\ &= \log_{10} E(\lambda\theta_2) - \log_{10} E(\lambda\theta_1) \\ &= 0.4343 C_2 (1/\lambda_1 - 1/\lambda)(1/\theta_2 - 1/\theta_1). \end{aligned} \quad (8)$$

If now it were possible to produce a colored glass in which the spectral transmission would exactly satisfy this equation, then such a glass would constitute a color filter which, if placed in front of a source whose distribution of radiant power corresponds to Wien's equation for one value of θ there is transmitted light whose spectral distribution also corresponds to Wien's equation, with a different and in this case higher value of θ.

It will be noted that Eq. (8) is of the first degree with respect to βt and $1/\lambda$, and can be plotted as a straight line if these values are used as coordinates. Fig. 2 is such a plot in which the values of $1/\lambda$ are plotted as abscissae and βt (with zero at the top) are plotted as ordinates. No absorption occurs—that is, $\beta t = 0$, at the point where $1/\lambda = 1/\lambda_1$. The slope is

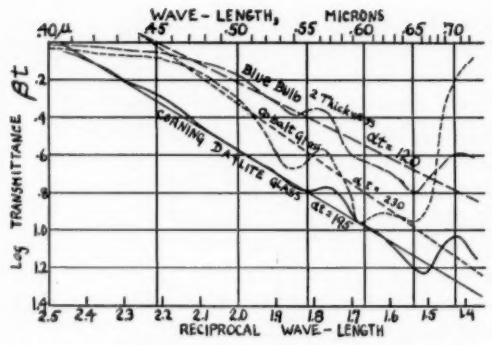

WAVE-LENGTH, MICRONS
.40μ .45 .50 .55 .60 .65 .70
Log TRANSMITTANCE βt
.2 .4 .6 .8 1.0 1.2 1.4
Blue Bulb 2 Thicknesses
Bausch & Lomb Glass
CORNING DAYLITE GLASS
2.5 2.4 2.3 2.2 2.1 2.0 1.9 1.8 1.7 1.6 1.5 1.4
RECIPROCAL WAVE-LENGTH

FIG. 2. Corning Daylite glass and other blue glasses showing the closeness of approximation of the curve of Daylite glass to a straight line when logarithm of transmittance (βt) is plotted against reciprocal of wave-length. The other blue glasses show irregularities and do not correct at wave-lengths less than 0.45μ. The Daylite glass illustrated raises the color temperature by 195 micro-reciprocal degrees, e.g., it raises the color temperature of a light source at 2848°K to the distribution corresponding to 6,400° if calculated by Wien's equation or 6,530°K if calculated according to Planck's equation.

$0.4343C_2(1/\theta_2-1/\theta_1)$. The position of λ_1 at which there is complete transmittance should be chosen so as to be just at the short wave-length end of the visible spectrum. If λ_1 is in the ultraviolet an excessive amount of absorption will be required throughout the rest of the spectrum, and if λ_1 is too far in the visible spectrum insufficient correction in the blue will result. It is thought that 0.41μ is a suitable wave-length to choose for λ_1. If the spectrophotometric measurements of an actual color filter show that βt plotted against $1/\lambda$ approximates a straight line, a color temperature altering filter is indicated and the closeness of approximation of the observed points to the straight line is a measure of the accuracy with which the filter gives the desired result.

From Eq. (8) it may be deduced that the same glass will serve to increase the color temperature of any source. The relation of the increased temperature of the source is such that[14] the difference in reciprocal temperatures in accordance with the Wien formula is a constant; i.e., $(1/\theta_1-1/\theta_2)=(1/\theta_3-1/\theta_4)$.

A variation in thickness of the glass (from t_1 to t_3) will leave β unchanged and the relation is:

$$\beta t_1/\beta t_3=t_1/t_3=(1/\theta_1-1/\theta_2)/(1/\theta_1-1/\theta_3).$$

That is, the differences in the reciprocal temperatures are in proportion to the glass thickness.

Relation of Formulae to Actual Glasses

Having deduced the mathematical formula to which a color filter must approximate with as great accuracy as possible, we will consider the application to an actual glass as measured on the spectrophotometer. Suppose the data for two carefully chosen wave-lengths λ_2 and λ_3 have the corresponding values of $\beta_2 t$ and $\beta_3 t$ as measured. Substituting these in turn in Eq. (8) and subtracting we get

$$\beta_2 t-\beta_3 t=0.4343C_2(-1/\lambda_2+1/\lambda_3)(1/\theta_2-1/\theta_1). \quad (9)$$

For the value $0.4343C_2$ its numerical equivalent 6223 may be substituted or it can be expressed in its reciprocal form $1/0.4343C_2=0.0001607$.

Also, $(1/\theta_2-1/\theta_1)$ is negative and the signs may be rearranged to give

$$\beta_2 t-\beta_3 t=6223(1/\lambda_2-1/\lambda_3)(1/\theta_1-1/\theta_2), \quad (10)$$

$$\frac{1}{\theta_1}-\frac{1}{\theta_2}=0.0001607\frac{\beta_2 t-\beta_3 t}{1/\lambda_2-1/\lambda_3}, \quad (11)$$

perhaps more conveniently expressed as

$$\Delta(1/\theta)=0.0001607\Delta\beta t/\Delta(1/\lambda). \quad (12)$$

Substituting αt for $\Delta(1/\theta)$

$$\alpha=0.0001607\Delta\beta/\Delta(1/\lambda). \quad (13)$$

As an example of measurements on an actual glass, a 5 mm piece of Corning Daylite glass made 7/13/29 gives

Point	λ	$1/\lambda$	βt	$\alpha t=0.0001607\frac{\Delta\beta t}{\Delta 1/\lambda}$,
(2)	0.42μ	2.381	−0.108	$=0.0001607\frac{0.845}{0.768}$,
(3)	0.62μ	1.613	−0.953	$=0.0001768$,
Difference Δ (2)−(3)		+0.768	+0.845	$\alpha=0.0000354$.

Wien Equation Passing through Two Given Points

To a close approximation any energy distribution expressible according to Planck's formula within the narrow range of the visible spectrum

[14] H. P. Gage, *Comments on Hyde and Forsythe Paper*, Trans. I. E. S. **16**, 428 (1921).

can also be expressed by Wien's formula by a suitable but different choice of θ. This follows from the fact that the curves for the two formulae are much alike (see Fig. 3). While the difference between the curves becomes considerable when working over the complete spectrum, the difference is not great over a narrow range. If for example the radiant power E_λ at two wave-lengths λ_1 and λ_2 is known, then either a Planck's equation curve can be drawn through these two points or a Wien's equation curve. The general slope of the two curves will determine the value of θ to use; the curvature will depend on which equation is chosen.

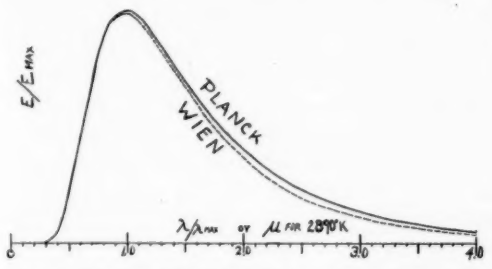


FIG. 3. Wien and Planck equation curves plotted with $E/E_{max.}$ against $\lambda/\lambda_{max.}$. When θ is 2890°, $\lambda_{max.}$ is 1μ.[8] The difference between the two curves is due to the Planck correction factor. The two curves have much the same shape, the divergence between them being gradual. The difference is negligible on the short wave-length side of the maximum. The Planck curve was plotted with $E/E_{max.}=1$ when $\lambda/\lambda_{max.}$ is 1. The Wien curve calculated with the same value of C_1 and θ is lower than the Planck curve at $\lambda/\lambda_{max.}=1$ by 0.7 percent.

Two points, such as those for 0.45μ and 0.65μ, can be selected from a series of Planck equation curves, such as those published by Frehafer and Snow,[8] Davis and Gibson[9] or by Fowle[7] and a Wien equation curve calculated which will pass through these same points. The divergence of all other points is remarkably small and is much less than the divergence from either curve of measured points on such high temperature sources as the sun, blue sky, etc.

The calculation of a Wien equation curve passing through two points (E_1, λ_1) and (E_2, λ_2) follows by substituting these values in Eq. (2) and solving for θ. Thus

$$\log_{10} E_1-\log_{10} E_2=5(\log_{10}(1/\lambda_1)-\log_{10}(1/\lambda_2))-(0.4343C_2/\theta)(1/\lambda_1-1/\lambda_2), \quad (14)$$

$$1/\theta=[5(\log_{10}(1/\lambda_1)-\log_{10}(1/\lambda_2))-(\log_{10} E_1-\log_{10} E_2)]/[0.4343C_2(1/\lambda_1-1/\lambda_2)]. \quad (15)$$

By substituting the numerical value for $0.4343C_2$ and inverting,

$$\theta=6223(1/\lambda_1-1/\lambda_2)/[5(\log_{10}(1/\lambda_1)-\log_{10}(1/\lambda_2))-(\log_{10} E_1-\log_{10} E_2)]. \quad (16)$$

Example. The latest data for sunshine outside the earth's atmosphere as given by Abbot and Fowle[15] have a slightly wavy curve. By choosing the crests of two such waves as indicative of the slope of the curve we get two points

	λ	$1/\lambda$	$\log(1/\lambda)$	E	$\log E$
Point 1	0.4753	2.104	0.32303	566	2.7528
Point 2	0.6858	1.458	0.16380	409	2.6117
Diff. (1)−(2)		+0.646	+0.15923		+0.1411

$$\theta=\frac{6223\times0.646}{5\times0.15923-0.1411}=\frac{4020}{6550}=6140°. \quad (17)$$

As shown later this corresponds to a value of θ of 6250° when used in the Planck formula. (See Fig. 4.)

In a similar way calculations were made with the points for 0.45μ and 0.65μ from the Planck equation data given in the Davis-Gibson paper[9] for each of the higher temperatures with the results shown in Table I.

For the two wave-lengths 0.45μ and 0.65μ the formula reduces to

$$\theta_{(Wien)}=4255/[0.7980-(\log E_1-\log E_2)]. \quad (18)$$

For such sources as the sun, sky, etc., the value θ serves as a convenient method of expressing a type of radiant power distribution. It may be permitted in this discussion, therefore, to use Wien's formula to express distributions repre-

[15] C. G. Abbot, F. E. Fowle and L. B. Aldrich, *The Distribution of Energy in the Spectra of the Sun and Stars*, Smithsonian Misc. Collections **74**, No. 7, 1–30 (1923).

BOSTON UNIVERSITY
COLLEGE OF LIBERAL ARTS
LIBRARY

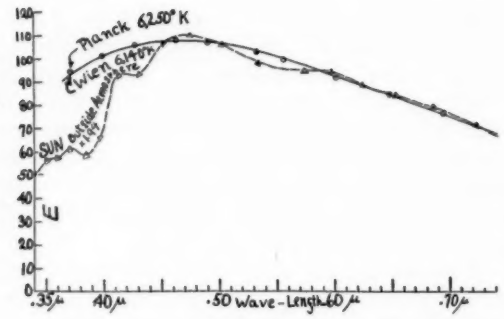


FIG. 4. Sun outside the earth's atmosphere, Abbot and Fowle, 1923 data[15] values multiplied by 1.94. Solid curve Wien distribution for 6140°, circles Planck's equation for 6250°, all curves adjusted to pass through common points at 0.454μ and 0.65μ. Within the visible region between 0.4μ and 0.7μ the difference between the two equations is negligible compared to the difference between either and the curve for the sun to which they are nearly equivalent.

senting stimuli approximating the various phases of daylight as well as other light sources whenever experiment justifies such use. Some other term than temperature might even be applied to the value of θ in these equations were it not for the fact that some incandescent sources not only are similar to, but actually are complete radiators whose actual temperature may be measured. It is to be pointed out, however, that with any experimentally obtainable incandescent sources temperatures exceeding 3780°K (crater of carbon arc) do not occur, and for such sources the correction factor changing the Wien to Planck equation within the visible spectrum is negligible, being but 0.5 percent at 0.7μ for 3780°K.

TABLE I. *Equivalence of Wien and Planck temperatures by red-blue ratio.*

Color temp. (°K)			Micro-reciprocal degrees		
Planck	Wien	Difference	$1/P$	$1/W$	Difference
5,000	4,990	10	200	200.4	0.4
5,500	5,460	40	181.8	183.2	1.4
6,000	5,920	80	166.7	168.7	2.0
6,500	6,390	110	153.8	156.5	2.7
7,000	6,840	160	142.9	146.0	3.1
8,000	7,710	290	125.0	129.7	4.7
9,000	8,520	480	111.0	117.4	6.4
10,000	9,280	720	100.0	107.8	7.8
12,000	10.680	1,320	83.3	93.7	10.4
14,000	12,005	1,995	71.4	83.3	11.9
16,000	12,970	3,030	62.5	77.1	14.6
18,000	13,900	4,100	55.6	71.9	16.3
20,000	14,650	5,350	50.0	68.2	18.2

COLOR TEMPERATURE OF TYPICAL LIGHT SOURCES

Ives[1] pointed out that the spectral energy distribution of the noon sun approximated that of a black body at 5,000° absolute.

The color temperature of various artificial illuminants was determined by Hyde, Forsythe and Cady and tables giving the resulting values have been published from time to time.[16, 17, 18] The color temperature of the carbon arc[19, 20] and some phases of daylight[21] were measured by I. G. Priest using the method of rotatory dispersion[22] and a preliminary standard of artificial sunlight, the "Abbot-Priest Sunlight"[23] was proposed. Another standard is average noon sunlight as shown in charts Nos. 1 to 20 of the Davis-Gibson filters.[9] According to Davis[24] the correlated color temperature of this distribution comes out 5394°K (Planck) as compared to the spectral centroid method of Priest[22] which gives 5578°. In addition to the above sources of information, summaries of these determinations are published in the *Report of the Colorimetry Committee*, L. T. Troland, Chairman,[25] by Norman Macbeth,[26] and

[16] Hyde, Forsythe and Cady, *Color Temperature Scales for Tungsten and Carbon*, Phys. Rev. **10**, 401 (1917), Table I.

[17] E. P. Hyde and W. E. Forsythe, *Color Temperature of Various Illuminants*, (Abs.) J. O. S. A. **1**, 99 (1917); J. Frank. Inst. 183, 353–354 (1917).

[18] E. P. Hyde and W. E. Forsythe, *Color Temperature and Brightness of Various Illuminants*, Trans. I. E. S. **16**, 419 (1921).

[19] I. G. Priest, *Measurement of the Color Temperature of the more Efficient Artificial Light Sources by the Method of Rotatory Dispersion*, J. O. S. A. **6**, 27–41 (1922).

[20] I. G. Priest, *Measurements of the Color Temperature of the more Efficient Artificial Light Sources by the Method of Rotatory Dispersion*, Bur. Stand. Sci. Pap. No. 443 (1922).

[21] I. G. Priest, *Color Temperature of Various Phases of Daylight*, J. O. S. A. **7**, 78 (1923).

[22] I. G. Priest, *The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion*, J. O. S. A. and R. S. I. **7**, 1175–1209 (1923).

[23] I. G. Priest, *Standard Artificial Sunlight for Colorimetric Purposes*, J. O. S. A. and R. S. I. **12**, 480 (1926).

[24] Raymond Davis, *A Correlated Color Temperature for Illuminants, Bur. Standards J. Research* **7**, 659–681 (1931), Research Paper 365.

[25] L. T. Troland, Chairman, *Report of Committee on Colorimetry*, J. O. S. A. and R. S. I. **6**, 527–596 (1922).

[26] Norman Macbeth, *Color Temperature Classification of Natural and Artificial Illuminants*, Trans. I. E. S. **23**, 302–324 (1928), Figs. 1 and 2

TABLE IIa. *Color temperature of selected light sources.* I.C.T., V. 247.

Source	Color temp. (°K) θ	Micro-reciprocal degrees $1/\theta$
Signal lamp, kerosene	1900	526.3
Hefner lamp	1880	531.9
Paraffin candle	1925	519.6
Kerosene—flat	2055	486.6
Acetylene, whole flame	2380	420.2
" Mees burner	2360	423.7
Vacuum lamps, carbon		
4Wpc carbon	2080	480.8
3.1 treated	2165	462
2.5 Gem	2195	455.5
Osmium —2 Wpc	2185	457.7
Tantalum—2 "	2260	442.5
Tungsten, 10 watt	2390	418.4
Standard value	2360	423.7
Straight filament, 25 watt	2493	401
40 "	2504	399
60 "	2509	398
Gas-filled tungsten, 75 watt	2705	369.6
100 "	2740	365
200 "	2810	355.9
Standard	2848	351.1
Movie, 900 "	3220	310.6
30,000 "	3300	303.0
Electric arc, +crater		
Solid carbon	3780	264.5
Cored carbon	3420	292.4

Source	Temp. (°C)	Color temp. (°K)	Reciprocal temp.
Complete radiator at the freezing point of			
Gold[28]	1063	1336	748.5
Palladium[28]	1555	1828	547
Platinum[10]	1773.5	2046.5	488.6
Rhodium[28]	1955	2228	449
Iridium[28]	2350	2623	381.3

in the *International Critical Tables*.[27] It was from the latter source that most of the data for Table II were taken except where indicated.

The sun, outside the earth's atmosphere, according to the data of Abbot given to the Bureau of Standards in 1917 (see Davis and Gibson,[9] Table I, p. 16), gives a correlated color temperature of 6565°K or a spectral centroid corresponding to 6724°K. If, however, it is plotted on a curve for 6500° Planck or 6390° Wien there is a close agreement of all points for wave-lengths greater than 0.45μ. The 1923 data of Abbot[15] come to a close agreement to the curve for 6250° Planck or 6140° Wien as has been previously described and as illustrated in Fig. 4. This method is, however, not considered rigorous for determining color temperature.

[27] *Temperature, Brightness and Efficiency of Selected Sources of Illumination*, Int. Crit. Tab. 5, 247, Table VII.

TABLE IIb. *Color temperature of various phases of daylight.*[21]

	Planck	$1/P$	Wien	$1/W$
Extremely blue sky	24,150	41.4	15,800	63.3
Blue sky range	19,000	52.6	14,000	70
	14,000	71.4	11,850	84.4
Whole sky	9,800	102	9,340	107
	6,000	166.7	5,900	169.5
Sun	5,300	188.7	5,260	190
	4,510	221.7	4,510	221.7
Sun outside earth's atmosphere, 1923 value of Abbot[15] and as illustrated in Fig. 4	6,250	160	6,140	162.9
Ditto, 1917 value, correlated color temperature, Davis[24]	6,565	152.3	6,445	155.2
Mean noon sun, correlated color temperature[24]	5,394	185.5	5,360	186.6
Abbot-Priest sun, correlated color temperature[24]	5,229	191.6	5,200	192.3

Complete radiators at the temperature of the freezing point of gold, palladium, platinum, have been used as standards for radiant energy. The freezing point of gold and palladium as given in the *International Critical Tables*[28] agree with those used by Forsythe[11, 18] and Roeser, Caldwell and Wensel[10] in their work on the freezing point of platinum but a new value for the freezing point of platinum, 1773.5°C, or 2046.5°K, has been determined and it is this temperature which is used in the discussion of the Waidner-Burgess standard of light.[29] The freezing points of rhodium and irridium might similarly be used if satisfactory refractories can be found. These latter temperatures are as given in the critical tables.

COLOR OF TRANSMITTED LIGHT FOR THICK PIECES OF DAYLITE GLASS

It has been noted that a thickness of Daylite glass may be found such as to reduce the reciprocal color temperature of a source to zero, and increased thicknesses will reduce it to less than zero. In this way chromaticities may be produced

[28] Int. Crit. Tab. 1, 103–104.

[29] H. T. Wensel, Wm. F. Roeser, L. E. Barbrow and F. R. Caldwell, *The Waidner-Burgess Standard of Light*, Bur. Stand. Res. Pap. 325, 6, 1103–1117 (1931).

corresponding to the energy distributions defined by Wien's equation for infinite and also for negative temperature. Although no physical meaning is to be attached either to infinite or to negative temperature, the Wien equation does give the actual distribution of the transmitted light. For example, 10 mm of the melt of Daylite glass of 7-13-29 changes the color temperature by an amount corresponding to 353 micro-reciprocal degrees, and 353 micro-reciprocal degrees corresponds closely to the standard gas-filled tungsten lamp. The spectral distribution resulting in such a case may be determined by substituting, $1/\theta=0$, in Wien's equation: $E_\theta=C_1/\lambda^5$, that is, the radiant power is inversely proportional to the fifth power of the wave-length.

One might readily suppose the interpretational difficulties to be a consequence of the inapplicability of the Wien equation to very high temperatures. Similar difficulties, however, appear with the Planck equation which is perfectly applicable by reason of its agreement with experimental data. For example, the Planck equation indicates that a radiator at infinite temperature would yield an energy distribution inversely proportional to the fourth power of the wave-length. This corresponds to a chromaticity even less blue than that indicated by the Wien equation for infinite temperature; and this chromaticity can, of course, also easily be obtained by means of Daylite glass.

Acknowledgment

The author wishes to acknowledge the helpful suggestions made by Dr. W. E. Forsythe on an early draft of the present paper.

Special acknowledgment is due the late Irwin G. Priest for stimulating the revision of a 1921 laboratory report into suitable form for publication. During the early part of this year Mr. Priest's letters to the author revealing his clear-cut conception of colorimetric problems expressed in his concise and exact diction were largely directed towards the simplification of the methods of using color temperature as a tool in expressing the characteristics of heterogeneous radiation caused by temperature. At least one of these letters dealing with an allied use of his rotatory dispersion method is worthy of amplification into a special article.

During my visit to Washington in May 1932 Mr. Priest discussed rather extensively and in considerable detail the advantages of using the reciprocal of color temperature as representing a more evenly graduated scale than do the temperatures themselves. Moreover, in the derivation of the radiation equations, temperature enters in as the reciprocal and, except for the fortuitous way in which the first thermometers were graduated, has as much reason to be considered fundamental. Mr. Priest therefore desired to express color temperature in the reciprocal form, using as a unit the microreciprocal degree, resulting in a scale in which 50 to 500 micro-reciprocal degrees covers the useful range from blue sky to candle light.[30] He was not ready to accept without further consideration the use of this scale based on the Wien equation rather than the Planck equation although the advantages to be gained in calculations involving thickness of color filters had been brought to his attention by Mr. Guild of the National Physical Laboratory in England. The purpose of the suggestion could he said, be equally well served by a table showing the differences between the two scales. A letter written by him on the last day of his life was a request for a filter to raise the color temperature of a light source by approximately 75 microreciprocal degrees and a hope expressed to discuss the matter further.

Acknowledgment is also given to Dr. Gibson and Dr. Judd of the Bureau of Standards for their careful study and suggestions regarding this paper, and to help received from several others as well.

[30] I. G. Priest, *A Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight*, J. O. S. A. **23**, 41 (1933).

Line Intensity and Energy Distribution in High and Low-Pressure Mercury Arcs

L. B. JOHNSON AND L. BURNS, *Radiation Measurements Laboratory, Massachusetts Institute of Technology*

(Received December 7, 1932)

INVESTIGATIONS of the intensity variations of mercury arc lines with operating conditions made by Küch and Retschinsky[1] and confirmed by Pflüger[2] and Athanasiu[3] show that lines which are members of the same series increase in intensity together. The above workers varied only the power input and made no effort to separate the effects of current and voltage. Harrison and Forbes in their articles on *Mercury Arc Characteristics*[4] and *Constricted Mercury Vapor Lamps*,[5] and Forbes and Leighton[6] on *Line Intensity as a Function of Mercury Vapor Pressure*, found that above a limited voltage, depending on the spectrum line and the input current, the intensity of the lines increased linearly. They also found that the ratio of intensities of any two lines of the same series was essentially constant.

Except for the work of Forsyth, Barnes and Easley[7] which led to the calibration of a spectroradiometer and data on the S_1 lamp, no data have been given on absolute intensities of mercury arc lines.

The method used to obtain absolute intensities in this research was one of photographic photometry with the use of a standardized mercury arc. This method of Stockbarger and Johnson[8] is one which is readily applied to both weak and strong sources.

The variation of the line intensity with current or voltage was measured on a quartz spectroradiometer. The absolute intensities of the lines were determined for only one current or voltage by photographic photometry. By applying the data from the spectroradiometer to the photometry results, curves of the absolute intensity variation with current or voltage may be drawn.

In the photographic photometry the spectrum lines photographed with the Hilger E_2 quartz spectrograph were broad and flat topped. Broad flat topped lines are of advantage in photometry since less care is needed in focussing the recording microphotometer to obtain reproducible results, than when fine lines are used. When fine lines are used, limitations in the resolving power of the microphotometer require that great care must be exercised in adjusting to obtain results which may be reproduced, since any unevenness in the glass of the plate or the emulsion introduces a considerable error. In the trial plates fine and broad lines gave identical results.

The standard arc, the absolute intensities of the lines of which had already been determined, was held in an asbestos board house having a one inch aperture. The arc was placed fifty centimeters behind a quartz diffusing disk ground on both sides. Wire screens used to cut down the intensity were placed between the diffusing disk and the arc. An image of the disk was cast two meters on the slit of the spectrograph by a quartz lens placed midway between the two.

Characteristic curves of the Eastman Single Coated Orthonon Plates, which were used throughout the research, were taken for each line to be measured, with the standard arc running at the current and voltage for which the line intensities were known.[8] The wire screens were used here to cut down the intensity of the light falling on the quartz plate. On the same plate with the characteristic curves, spectrograms of other sources were also taken. The plate was then given a standard development in Eastman D61A developer. The densities of the lines were then recorded by the microphotometer and those of the standard were plotted on log log paper, against the known intensities incident on the

[1] Küch and Retschinsky, Ann. d. Physik **20**, 563 (1906); **22**, 595 (1907); **22**, 852 (1907).

[2] Pflüger, Ann. d. Physik **26**, 789 (1908).

[3] Athanasiu, Comptes Rendus **178**, 2071 (1924).

[4] Harrison and Forbes, J. Opt. Soc. Am. **10**, 1 (1925).

[5] Forbes and Harrison, J. Opt. Soc. Am. **11**, 99 (1925).

[6] Forbes and Leighton, J. Opt. Soc. Am. **12**, 53 (1926).

[7] Forsyth, Barnes and Easley, J. Opt. Soc. Am. **21**, 30 (1931).

[8] Stockbarger and Johnson, not yet published.

quartz plate as abscissae, and the characteristic curves drawn. The density of a line in the unknown source was then spotted on the characteristic curve of the same line from the standard arc. The reading from the abscissa gave the intensity of the line in the unknown source.

The wire screens used to cut down the intensity of the light falling on the quartz disk were carefully constructed by soldering together, at the edges only, pieces of screen one on top of another so that a definite pattern was formed when looking through the screens, and no two wires of different screens ran parallel. Their transmissions were determined with a thermopile and galvanometer. For each exposure taken on the plate only one such prepared screen was used kept in constant random motion.

As a check on the method of using screens, the inverse square law was used, the arc being moved away from the quartz plate. The soundness of the use of this law in this case had been proven by a linear thermopile and galvanometer.

In the spectroradiometric measurements it was impossible to separate the yellow, blue, violet, and the group of lines at about 3660 into their components because of the poor resolving power, and the necessity of using a fairly wide slit to obtain sufficient intensity. In the calibration, however, the intensity of the components is known and recorded below. At all other points than the calibration points the intensities of groups are given.

The High-Pressure Arc

The high-pressure arc used had an arc length of three inches and was of the vertical type. The voltage across this arc was regulated by a specially built photoelectric device, which, by means of a voltmeter, a photo-cell, and an amplifier, controlled a blast of air on the cathode of the arc. The current was kept constant at 4 amperes by a hand-controlled rheostat.

The intensities of the visible and near ultraviolet lines when this arc was operated at 45 volts and 4 amperes were as given in Table I.

Fig. 1 shows the variation of the intensities of the line groups with arc voltage. The arc current is 4 amperes. The lines 5461, 4358, and 4047 belong to the same series and thus should increase in intensity together. The intensity ratio for any two lines in the series should be essentially constant. This is found to be true when applied to the groups of lines. From the table above it is obvious that the weaker components of a group add little to the group itself and thus the behavior of the group may be considered to be that of the strongest line. Thus the groups may be considered to behave as the lines of the series.

Table I.

Group	Line	Intensity (watts $\times 10^{-6}$ at 50 cm for 1 in. of arc)
Yellow	5791	0.102
	5770	0.124
Green	5461	0.305
Blue	4358	0.232
	4348	0.014
	4339	0.006
Violet	4078	0.022
	4047	0.170
3660	3663	0.075
	3655	0.086
	3650	0.190

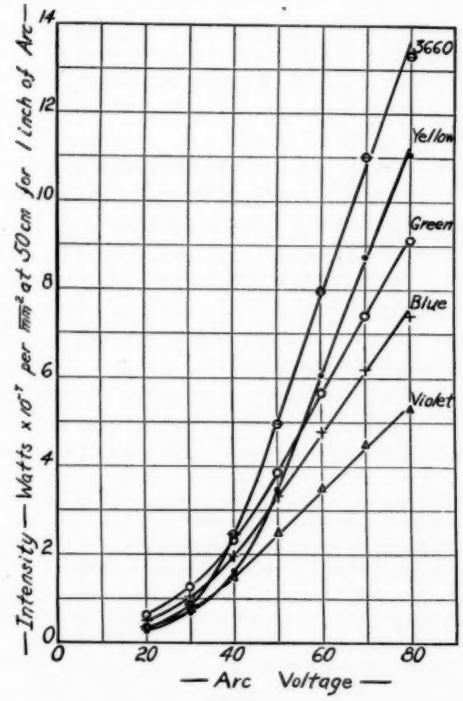


Fig. 1.

The analysis of the radiation was carried out by filter radiometry. The radiation was divided into four groups.

Group	Region	Group	Region
A	14,000 to 40,000A	C	2,950 to 3,800A
B	3,800 to 14,000	D	1,850 to 2,950

An analysis of radiation constitutes the measuring of the intensities of the several regions as a percentage of the total intensity.

A copper-constantan thermopile known to be nonselective and a Leeds and Northrup low-resistance, high sensitivity galvanometer employing a scale distance of three and one-half meters were used in the analyses and intensity measurements. Whenever possible another similar galvanometer with a shorter scale distance was used to give a check on the results. The long scale distance galvanometer was checked for linearity by a calibrated voltage box. Measurements of absolute intensity were made with a standard incandescent lamp checked against a primary calibrated at the Bureau of Standards.

The analyses were made with three kinds of filters, Noviol O glass, G86B glass, and a one centimeter quartz water-cell, the procedure being to compare the intensity of the filtered radiation

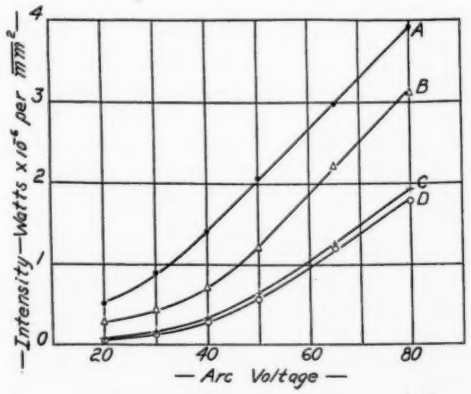


FIG. 2.

to the total. The cut-offs of the filters were known since they are in constant use in the laboratory.[9] Both glasses have high transmissions in the infrared well beyond the cut-off of the water-cell so that when used with the latter their infrared absorption need not be considered. For each region two identical filters were used to estimate the filter transmission, by determining the ratio of the intensities of radiation transmitted by two filters and by one filter, respectively. Since the filter cut-offs were not vertical the ratio had to be

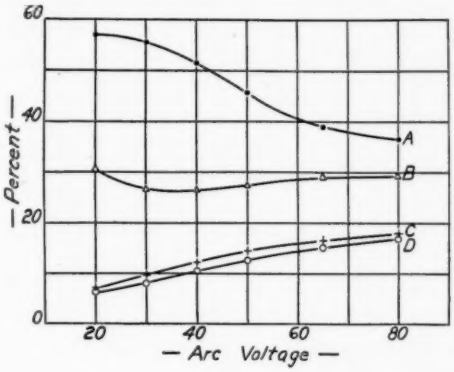


FIG. 3.

determined for each running condition of the arc, because of the change in energy distribution.

Fig. 2 shows the variation of the intensity of these regions with arc voltage. The intensities are given in watts $\times 10^{-6}$ per mm² at 50 cm for the whole arc, anode and cathode being included.

The total intensity varied as follows:

Arc voltage	80	70	60	50	40	30	20
Intensity (watts $\times 10^{-6}$/mm² at 50 cm)	10.74	8.60	6.50	4.43	2.72	1.57	0.888

Fig. 3 shows the variation of the analysis with arc voltage and an arc current of 4 amperes. This analysis is again for the entire arc, the anode and cathode included. The ordinate of the curve is percent of the total intensity.

LOW-PRESSURE ARCS

For a low-pressure quartz mercury arc 22 inches long and running at 4 amperes and about 37 volts the intensities of the lines in the visible and near ultraviolet are given in Table II.

Fig. 4 shows the variation of the intensities of the groups as the arc current is varied keeping the voltage approximately constant at 37 volts.

Again it will be noticed that the weak lines in a group have little effect upon the variation of the group and thus 5461, 4358, and 4047 may be

[9] Stockbarger, J. Opt. Soc. Am. **14**, 356 (1927).

TABLE II.

Group	Line	Intensity (watts/mm² at 50 cm for 1 in. of arc)
A Yellow	5791	0.009
	5770	0.011
B Green	5461	0.057
C Blue	4358	0.053
	4348	0.0008
	4339	0.0003
D Violet	4078	0.0058
	4047	0.042
E 3660	3663	0.0068
	3655	0.0075
	3650	0.0126

said to increase together and that the intensity ratio of any two remains essentially constant.

Great difficulty was encountered with this arc, both on the line intensity work and the analyses, since the arc was extremely sensitive to drafts. It was difficult to obtain check results on two different days because of changes in room temperature. Two other effects were also noted. When the arc current was suddenly reduced, immediately the total intensity went up, but after 10 or 15 minutes came back to approximately the same value.

It was found when obtaining the intensity relations of lines that the arc had a hot and a cold

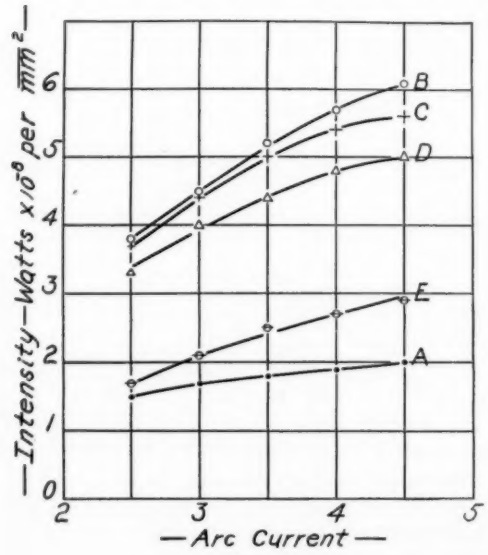


FIG. 4.

"side." That is, if the arc was running at say, 40 volts, and a blast of air was played upon it, the arc voltage would decrease until a minimum was reached at about 36 volts. At this point if the arc was further cooled the voltage would again increase. It was found qualitatively that the line intensity depended on whether the arc was running on the hot or the cold side of the minimum. The data given in this paper are for the arc running on the hot side, that is, if the arc had been cooled the voltage would have gone down.

Fig. 5 shows the variation of the analysis at 1 meter for 5″ of low-pressure quartz arc with arc current. It will be noticed that the curve *E* of the total intensity is drawn as a straight horizontal line. As stated above, the arc was extremely sensitive to changes of air temperature and it is believed that the failure of the points to fall on the line or to show a decrease in intensity was due to varying ventilation. A characteristic, which also leads to the belief that the intensity is essentially constant, is, that although the arc appears much less brilliant when running at 2.5 amperes than at 4.5 amperes, it is noticed that the quantity of ozone produced is tremendously increased, which presumably means an increase of short wave-length radiation. A spectrogram taken with a small quartz Hilger spectrograph shows the presence of strong lines at 2537 and below to 1849, the latter showed up well on an oiled plate without overexposing the plate to radiation of longer wave-length.

Table III gives the analysis for various arc currents. The shape of the curves of the anal-

TABLE III.

Region	4.5a	4.0a	3.5a	3.0a	2.5a
14,000 to 40,000	12.3%	13.0%	11.9%	11.0%	9.9%
3,800 to 14,000	53.9	52.8	50.0	45.0	37.8
2,950 to 3,800	15.0	13.4	12.0	10.3	8.8
1,850 to 2,950	18.8	20.8	26.1	33.7	43.5

ysis are of course the same as Fig. 2 since the total intensity was assumed constant.

The intensity of the far ultraviolet reaches the high value of 43.5 percent at 2.5 amperes. Qualitative results show that if the current is further reduced, still higher percentages are found. Below 2.5 amperes the arc is very unstable and goes

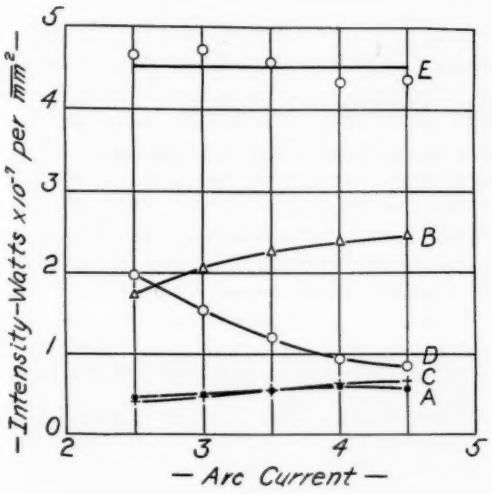


FIG. 5.

out frequently, making it impossible to obtain quantitative results at lower currents.

To really study the variation of the line intensities of such a low-pressure arc there should be a superior temperature control. Such control might be obtained by running the arc in a ventilated box and accurately controlling the temperature of the air entering the box. In this way a study of the variation of line intensities on the hot and cold sides of the minimum might be made.

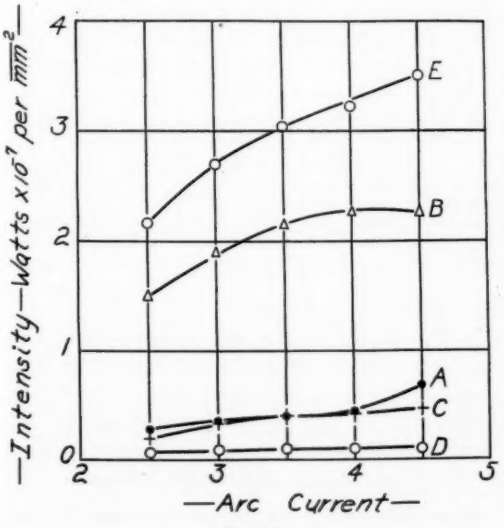


FIG. 6.

Fig. 6 shows the intensity variations for 5″ of arc at one meter of the same regions of a similar low-pressure arc, but in Uviol glass instead of quartz. The total intensity falls in this arc as the current is reduced, probably because the increasing far ultraviolet is cut off by the glass. Variations in the other regions from the results of the quartz arc are probably due to the heating effect of this absorbed ultraviolet radiation.

The analysis of the arc at different currents is given in Table IV.

Two other arcs were analyzed at 4.0 amperes only. One made of UV 690 glass gave the analysis shown in Table V.

The total intensity of this arc at 1 meter for 5″ of arc was 0.371 watts $\times 10^{-6}$ per mm².

An arc made of UV888 glass gave the analysis shown in Table VI.

The total intensity of this arc at 1 meter for 5″ of arc was 0.371 watts $\times 10^{-6}$ per mm².

In conclusion we are indebted to the General Electric Vapor Lamp Company for the loan of the arcs used. We are especially indebted to Dr. D. C. Stockbarger who gave us very helpful advice and under whose direction the work was done.

TABLE IV.

Region	4.5a	4.0a	3.5a	3.0a	2.5a
14,000 to 40,000	19.8%	14.0%	12.9%	13.8%	14.9%
3,800 to 14,000	64.1	69.9	71.2	70.1	68.9
2,950 to 3,800	13.2	13.1	12.9	13.1	13.1
1,850 to 2,950	2.9	3.0	3.0	3.0	3.1

TABLE V.

Region	4.0a	Intensity (watts $\times 10^{-6}$/mm² at 1 m for 5″ of arc)
14,000 to 40,000	16.1%	0.060
3,800 to 14,000	67.3	0.250
2,950 to 3,800	14.5	0.054
1,850 to 2,950	2.1	0.007

TABLE VI.

Region	4.0a	Intensity (watts $\times 10^{-6}$/mm² at 1 m)
14,000 to 40,000	15.0%	0.056
3,800 to 14,000	67.4	0.250
2,950 to 3,800	15.1	0.056
1,850 to 2,950	2.5	0.009

The Measurement of Erythemal Ultraviolet Radiation

A. H. TAYLOR, *Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland*

(Received October 19, 1932)

IN order to appraise the relative therapeutic outputs of various ultraviolet sources it is desirable to use some measuring device which weights the radiation in different spectral regions approximately in proportion to its therapeutic value. Unfortunately, we have no data establishing the relative effectiveness of different spectral regions in the maintenance of health and the cure of disease, hence this type of measurement is not practicable at present.

The relative effectiveness of different spectral regions in the production of erythema has been fairly well determined. Fig. 1[1] shows the results of five researches. While results obtained by different observers vary considerably, all agree in showing that λ2967 in the mercury spectrum is the most effective wave-length, per unit of energy, in producing erythema. Work done at Cornell University[2] indicates that this wave-length is also very important in the prevention and cure of rickets in chickens, and presumably the same would apply to rickets in children.

Erythema is very generally used by the medical profession as a guide to the proper dosage in therapeutic applications of sunlight and artificial sources of ultraviolet radiation. Furthermore, it is the criterion by which the public appraises the value of a source of ultraviolet radiation. Hence, if any device or method of measurement appraises different sources of ultraviolet radiation approximately in proportion to their erythemal power, it should be fairly satisfactory for the appraisal of such sources at the present time. If the spectral effectiveness of ultraviolet radiation for the cure of disease should later prove to be appreciably different from its erythemal effectiveness, the method of measurement could be revised. However, the difficulty of determining the former precludes any likelihood of an early need for a change of this nature.

During the past three years Lighting Research Laboratory has investigated many methods of measuring ultraviolet radiation.[3] These have included the use of various types of photoelectric cells, fluorescent glasses and the erythemal reaction of untanned human skin. Chemical tests were not included, since they require an appreciable exposure-time. Quickness of response is essential to a method to be used in routine testing, and very desirable in any testing method.

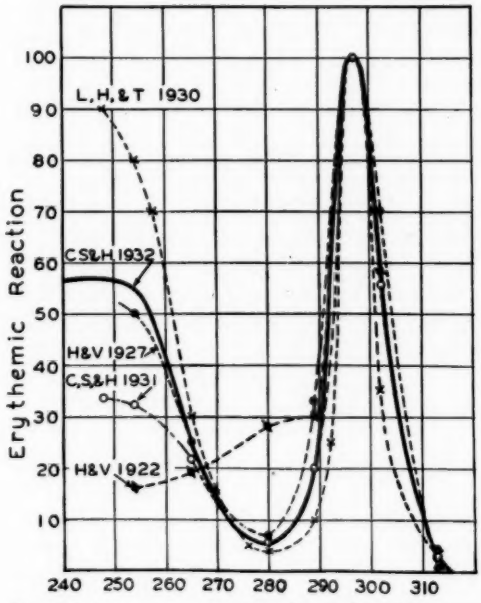


FIG. 1. Relative spectral erythemic reaction of the human skin to equal amounts of radiant energy at various wave-lengths; H. and V., Hausser and Vahle; L. H. and T., Luckiesh, Holladay and Taylor; C. S. and H., Coblentz, Stair and Hogue (from Bur. Standards J. Research **8**, 544 (1932).

[1] *The Spectral Erythemic Reaction of the Untanned Human Skin to Ultraviolet Radiation*, Coblentz, Stair and Hogue, Bur. Standards J. Research **8**, 541 (1932).

[2] *The Amount of Ultraviolet Radiation Necessary to Cure Rickets*, Maughan and Dye, J. O. S. A. **20**, 279 (1930).

[3] *Measurement of Biologically Important Ultraviolet Radiation*, Taylor and Holladay, Trans. I. E. S. **7**, 711 (1931).

Erythemal Tests

When making erythemal tests six or twelve ⅜-inch holes closely spaced are punched in a piece of gummed cloth tape 1¼ inches wide. This tape is pasted on some untanned portion of the subject's skin, e.g., the back, abdomen or arm. Exposures to the source being tested, at a measured distance or illumination, are made for varying lengths of time, the holes being covered successively after increasing exposures-times. Fig. 2

Fig. 2. Photograph of subject taking erythemal test on arm.

shows a test in progress. If two or more sources are being compared, the exposures are made on closely adjacent skin areas. The resultant erythema is graded on the following morning, after an interval of 16 to 20 hours. In this grading two objectives are aimed at, *viz.:* to determine the exposure which produces an MPE (minimum perceptible erythema), and exposures with different lamps which produce equal degrees of erythema. It is frequently found that there are several matched pairs of spots (of equal reddening) when two sources are compared. All of these are taken into consideration in arriving at a relative ultraviolet output rating for the two lamps.

The MPE rating is conservative, and in general it is applied to a spot which is well defined. Frequently a faint reddening is perceptible for some spot or spots which have been exposed for a shorter time, but unless the outline of the spot is fairly definite, it is not given an MPE rating. The maximum amount of reddening develops approximately five or six hours after the exposure, and after this time it gradually decreases. Hence the exposure required to produce an MPE rating depends to some extent upon the time which elapses between exposure and observation. However, it is found that when exposures are made to sunlight or Mazda Sunlight lamps, ratings at six and twenty hours after exposure are substantially the same.

The exposure required to produce an MPE varies with different individuals. Also, the same individual varies in his sensitivity at different times and upon different parts of the body.

We recently exposed eleven adult male subjects to three Type S-1 Mazda Sunlight lamps. The lamps were operated in a Model B Sunlamp reflector, axis vertical, the subjects being placed 30 inches from the plane of the reflector opening. The three lamps used had been selected from a large group of lamps and represented the extremes and an intermediate of the group, as measured by a photoelectric cell to be discussed later. Table I shows the exposure required to produce an MPE with each subject, and gives some idea as to the variations between individuals. Table II shows the relative energy ratings for the three lamps as derived from exposures required to produce equal degrees of erythema, by using all spots for which equal erythema was obtained. It indicates the accuracy to be expected in measuring the relative

Table I. *Exposure data for three Type S-1 Mazda Sunlight lamps operated in a Model B Sunlamp reflector, axis vertical at 110 volts.*

Subject	Minutes exposure required to produce an MPE		
	Lamp P-6	Lamp P-12	Lamp P-17
B	10.5	5.8	4.1
G	11.5	5.5	3.8
JD	9.5	5.2	3.3
CD	12.0	6.0	3.2
GK	13.3	7.0	4.5
E	....	3.5	2.6
TK	9.0	4.8	2.8
C	(16.0)	7.0	5.0
P	15.8	7.0	3.6
W	9.2	6.0	3.3
T	10.3	5.8	3.4
Average	11.2	5.8	3.6

TABLE II. *Relative erythemal ultraviolet ratings of three S-1 lamps as determined by exposures of eleven individuals.*

Subject	Relative exposure-time for equal erythema Lamp P-6	Lamp P-12	Lamp P-17
B	183	100	68
G	193	100	69
JD	179	100	64
CD	208	100	54
GK	191	100	64
E	...	100	67
TK	173	100	58
C	(220)	100	71
P	211	100	47
W	183	100	65
T	177	100	59
Average	193	100	62
*Weighted average	189	100	63
Relative erythemal ultraviolet output	53	100	159

* Based on number of paired erythema spots.

erythemal output of different lamps by this method.

PHOTOELECTRIC CELLS

The photoelectric cell appeals to everyone as being the most convenient device for measuring ultraviolet, because of its high sensitivity and quick response. Sodium cells in quartz or Corex D bulbs have been frequently used for this purpose. Fig. 3 shows the relative spectral sensitivities of several sodium cells obtained from different sources, together with the erythemal curve. For convenience in determining the suitability of such cells for the measurement of erythemal ultraviolet radiation, these curves have been plotted on a basis of 100 at λ2967 (wave-length in Angstroms), since the maximum of the erythemal effectiveness curve is at this point.

Numerous investigators[4, 5, 6, 7] have found cadmium photoelectric cells suitable for the measurement of erythemal ultraviolet radiation. Fig. 4 shows the relative spectral sensitivities of

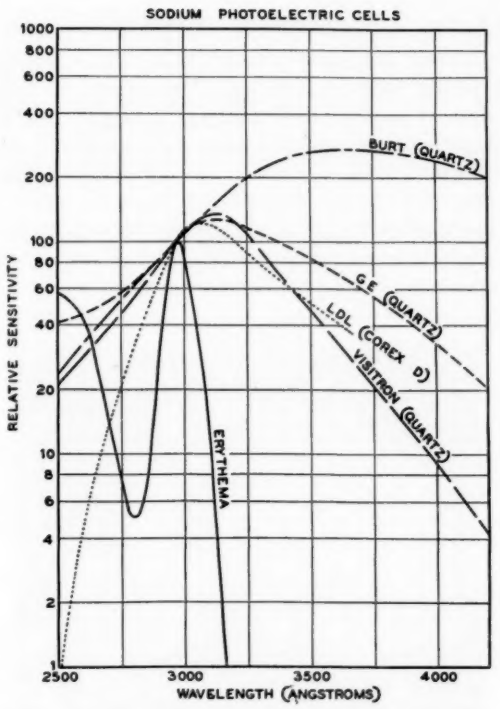


FIG. 3. Relative spectral sensitivities of various sodium photoelectric cells, also average erythemal curve.

three cadmium cells, two of these (Osram and G. E.) being in quartz bulbs and the third (L. D. L.) in a Corex D bulb. The last named was made by Dr. W. A. Roberts of the Lamp Development Laboratory at Nela Park, as such cells are not commercially available in the United States at present. This figure also shows the spectral sensitivity of a titanium photo-cell, with Corex D bulb, obtained from Dr. Rentschler of the Westinghouse Lamp Company.

FLUORESCENT GLASSES

There are many materials which fluoresce when exposed to ultraviolet radiation. We have investigated the spectral sensitivities of uranium glass and blue-fluorescing glass, both obtained from the Corning Glass Works. Since for both glasses the fluorescence for wave-lengths shorter than λ3200 takes place at or near the surface of the glass, the method of use is to bevel and polish one edge and view this edge (method described

[4] *Zum Gebrauch der Cadmiumzelle fur Messungen der ultravioletten Strahlung*, Dorno, Strahlentherapie 25, 351.

[5] *The Calibration of the Mercury Vapor Lamp in Reproducible Units for Clinical Purposes*, Pohle and Sawyer, Am. Jour. Roent. and Rad. Ther. **20**, 338 (1928).

[6] Uber Erfahrungen bei UV-Messungen mit der Cadmiumzelle, Ruttenauer, Strahlentherapie **31**, 349 (1929).

[7] *Ultraviolet Measurement with the Cadmium Photo-Electric Cell*, Schwarzschild, Am. Jour. Roent. **26**, 272 (1931).

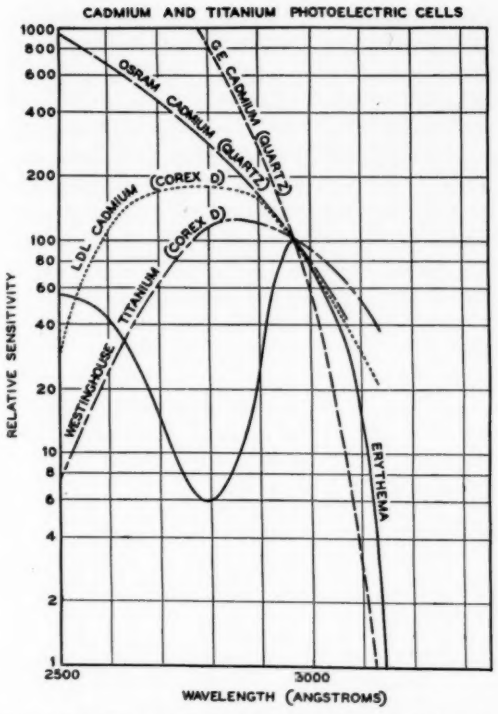


FIG. 4. Relative spectral sensitivities of three cadmium and one titanium photoelectric cell.

by Dr. H. P. Gage). When the proper face of the glass is exposed to the ultraviolet radiation the beveled edge glows with a yellow or blue light, depending upon which glass is used.

In the determination of the spectral sensitivities of these glasses they have been enclosed so that the fluorescent brightness was measured by

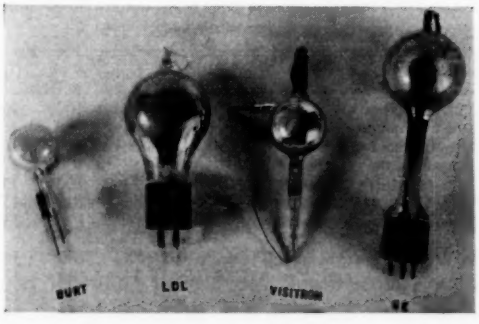


FIG. 5. Four types of sodium photoelectric cells.

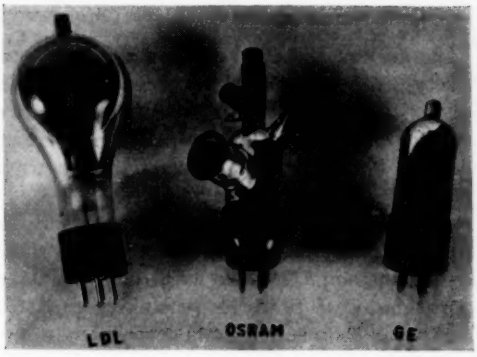


FIG. 6. Three cadmium photoelectric cells.

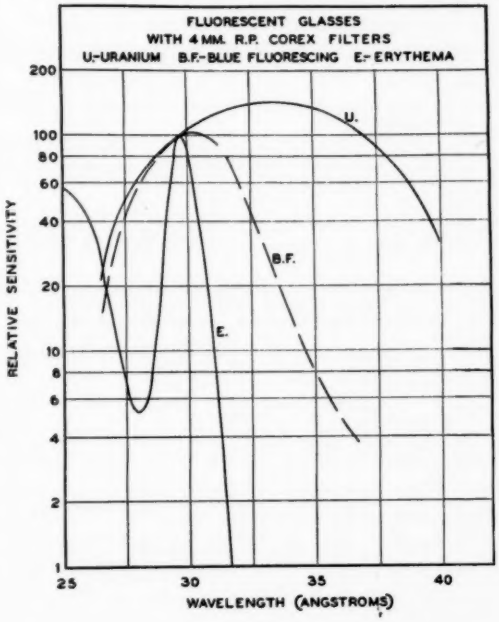


FIG. 7. Spectral sensitivities of fluorescent glasses with 4 mm red-purple Corex filters. (The wave-length values in this figure should be 2500, 3000, etc. Angstroms.)

means of a photoelectric cell sensitive to the color of the fluorescent light—a caesium-oxide cell for uranium glass and a potassium cell for the blue-fluorescing glass. The monochromatic energy incident on the glass was measured by means of a thermopile. Fig. 7 shows the spectral sensitivities of these glasses measured in this way, when 4 mm filters of red-purple Corex A glass over the fluorescent glass are used.

Discussion of Sensitivity Curves

The curve of spectral erythemal effectiveness is very steep on the long wave-length side. The sensitivity to λ2967 is approximately 35 times as great as the sensitivity to λ3132. Also, there appears to be no erythemal effect due to wave-lengths longer than approximately λ3200.

Of all the devices examined by us, only the cadmium cell is practically insensitive to radiation longer than λ3200. Its sensitivity to λ2967 is approximately 5 times its sensitivity to λ3132. Even though its sensitivity curve differs appreciably from the erythemal curve, a number of investigators have found it quite satisfactory in appraising different sources of the same type, e.g., quartz mercury lamps, for erythemal power. Our experience agrees with this, as will be shown later.

The titanium photo-cell tested would also appear to be fairly satisfactory for measurements of erythemal effectiveness, although its sensitivity to λ3132 is undesirably high.

It is possible partially to correct for sensitivity to wave-lengths longer than λ3200 by a "differential" method. An example of the application of this method is a sodium photoelectric cell exposed to the source with and without a piece of window-glass interposed. Fig. 8 shows the spectral sensitivity of a commercially-available quartz sodium cell when used with an opaque shutter and when used with a window-glass shutter. Although this method improves the sensitivity curve, it is still far from satisfactory as regards an approximation to the erythemal curve. The resultant sensitivity would be unsatisfactory when comparing sunlight with artificial sunlight lamps.

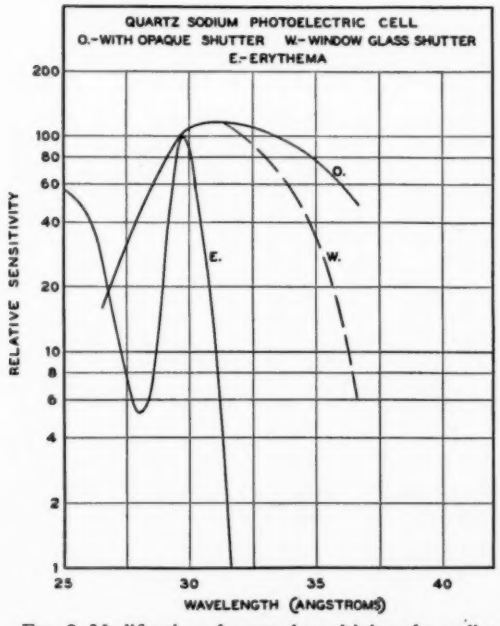


Fig. 8. Modification of spectral sensitivity of a sodium photoelectric cell by use of window-glass shutter. (The wave-length values in this figure should be 2500, 3000, etc. Angstroms.)

Appraisal of Erythemal Ultraviolet Radiation

In Table II are given the results of erythemal tests with three specially selected Type S-1 Mazda Sunlight lamps differing widely in ultraviolet output. During the tests ultraviolet energy measurements of these lamps were made with a blue-fluorescing attachment to the Macbeth illuminometer. Prior to the tests, measurements of these and ten other lamps were made with a cadmium photoelectric cell with quartz window, obtained from the Osram Company in London. Results with this cell were so favorable that Dr. W. A. Roberts of the Lamp Development Laboratory made up several gas-filled cadmium cells in Corex D bulbs, and measurements of the same lamps with these cells and two sodium cells in Corex D bulbs were made.

Table III shows the relative ultraviolet ratings of three lamps by various methods and devices. From this table it is apparent that the cadmium

Table III. *Relative ratings of three S-1 lamps in Model B Sunlamp reflector, axis vertical, as measured by different methods.*

	P-6	P-12	P-17
Erythemal tests, 11 subjects	53	100	159
Blue-fluorescing glass, visual	64	100	132
Blue-fluorescing glass, photo-cell	86	100	128
Corex sodium photo-cell No. 1	88	100	126
Corex sodium photo-cell No. 2	74	100	112
Ditto, window-glass shutter No. 1	85	100	130
Ditto, window-glass shutter No. 2	71	100	117
Quartz cadmium photo-cell (Osram)	55	100	144
Corex " " (LDL No. 3)	54	100	134
" " " (" " 4)	59	100	170
" " " (" " 5)	72	100	143
" " " (" " 6)	65	100	137

cell measurements agree with the erythemal test results more closely than measurements by any other method used. The blue-fluorescing glass attachment with the Macbeth illuminometer gave the next best results. The blue-fluorescing glass, when used with a red-purple Corex filter and a photoelectric cell to measure the fluorescent brightness, does not give results as favorable as those obtained by the visual measurement of the fluorescence. The reason for the difference is not entirely clear, but in part it appears to be due to the presence of stray radiations other than the fluorescent light. The sodium photo-cells with Corex D bulbs are not superior to the blue-fluorescing glass with photo-cell.

Any measuring device or method which is appreciably sensitive to radiation of wave-lengths longer than λ3200 is likely to give too high an appraisal to natural sunlight as compared with artificial sunlight lamps, if the purpose of the measurement is to determine their relative erythemal power. Conversely, any device or method which appraises natural sunlight and Mazda Sunlight lamps approximately in proportion to their erythemal power should be quite satisfactory for the measurement of various types of artificial sunlight sources such as Type S-1 and S-2 Mazda Sunlight lamp, Type G-1 lamp and Mazda CX lamp.

Table III shows that cadmium cell No. 4 gave results on three S-1 lamps very nearly in the same ratio as the erythemal tests. Further tests have been made with this cell, comparing sunlight and a Type S-1 lamp in a Model B Sunlamp reflector. Eight individuals were used, the exposures being made on the upper arm, front surface. Measurements of the ultraviolet intensity were made by use of cadmium cell No. 4 connected to a sensitive galvanometer. The erythemal results were compared in the manner previously described. Table IV shows the relative exposures (galvanometer deflection × minutes) for sunlight and for a Model B Sunlamp to produce equal amounts of erythema. Each value in this table is the average obtained from exposure ratios for two to six pairs of matched erythemal spots. Since there appeared to be some question whether the ratio obtained depended upon the elapsed time between exposure and observation, five subjects were examined approximately 5 and 21 hours after exposure. Although individuals varied, the average result for the group was the same at both times of observation. In general, it appeared that there was very little fading of the erythema in this time.

TABLE IV. *Summary of data comparing the appraisal of sunlight and a Model B Sunlamp by erythemal tests and energy measurements with a cadmium photo-cell.*

Subject	Ratio of exposures (ultraviolet*-minutes) for sunlight and for Model B Sunlamp to produce equal degrees of erythema	
	Observed on day of exposure	Observed next day
K		1.86
T		1.06
G		1.90
B		2.96
W		1.93
T (2nd test)		1.03
G " "	1.07	1.48
W " "	1.14	1.14
M	1.00	0.92
E	1.36	1.19
CE	1.91	1.83
		1.57

* Galvanometer deflection when cadmium photo-cell in Corex D bulb was exposed.

This cadmium cell has also been used to measure the erythemal radiation from Type S-2 Mazda Sunlight lamps and a glow lamp (Type G-1). The results were in very close agreement with erythemal tests of the same lamps.

Since the quartz mercury lamp is so generally used in therapeutic work by the medical profession, it was considered desirable to determine whether the relative appraisal of quartz mercury arcs and Mazda Sunlight lamps by this cadmium cell would agree with erythemal tests of the same lamps. The average results for three subjects showed that the two methods of appraisal agreed within about five percent. This agreement is better than had been expected, in view of the fact that this cell has a high sensitivity in the spectral region λ2600 to λ2900, where the erythemal effectiveness curve is low. However, the quartz mercury arc is low in energy output in the region from λ2700 to λ2900, hence this difference is not so important. It might be expected that if a source such as a carbon arc using therapeutic C carbons were measured, the cadmium cell rating would be too high because of a large amount of energy between λ2600 and λ2900.

The cadmium cell used in this test is only slightly sensitive to any radiation transmitted by ordinary window-glass, regardless of whether the source is sunlight or the S-1 lamp. For either source the apparent transmission of a piece of window-glass was found to be only 2 or 3 percent when measured by this cell.

The current obtained from a cadmium cell is much lower than that obtainable from a sodium cell, but cell No. 4 (gas-filled) has been found quite satisfactory when used with a Leeds and Northrup No. 2500b galvanometer, in the measurement of the ultraviolet energy in sunlight and the radiation from Type S-1 and S-2 Sunlamps. With a portable direct-current amplifier[8] measurements of the ultraviolet component of the radiation from a Mazda CX lamp (high-efficiency tungsten lamp in ultraviolet-transmitting bulb) has been measured, with an illumination of only 12 footcandles. With this illumination the energy for all radiations shorter than λ3150 is approximately 0.1 microwatt per square centimeter.

Summary

Various methods of appraising ultraviolet radiation have been studied. Many of these have been compared with exposures of untanned skin and the resultant erythema produced. Of all devices and methods studied, a cadmium photoelectric cell with Corex D bulb was found to give results most nearly in agreement with erythemal tests. It was found to give very satisfactory agreement when comparing Mazda Sunlight lamps Types S-1, S-2 and G-1, also a quartz mercury arc. When the cell was calibrated with a Type S-1 lamp which had been used in erythemal tests, and then used to measure natural sunlight, it was found that its appraisal of sunlight was approximately 50 percent higher than it was when erythemal tests were used. In view of the low accuracy obtainable with erythemal tests, and the large variations in erythemal sensitivity of different individuals, this degree of agreement appears satisfactory for testing artificial sunlight lamps at the present time.

The cadmium cell has a low sensitivity, and requires a sensitive galvanometer, electrometer, or some type of amplifier. A portable amplifier[8] for this purpose has been developed. Its calibration is reproducible from day to day with an accuracy as great as that obtainable with portable illuminometers.

[8] Taylor and Kerr, *A Direct-Current Amplifier with Good Operating Characteristics*, Rev. Sci. Inst. **4**, 28 (1933).

The Herschel Effect as a Result of Simultaneous Forward and Reverse Reactions

KARL M. JOEHNCK AND JULIAN M. BLAIR, *University of Colorado*

(Received August 3, 1932)

BLAIR and LEIGHTON[1] have suggested an equation for the rate of growth of the latent photographic image. That equation is based upon the hypothesis that the photographic reaction is dual in nature—the result of forward and reverse reactions operating simultaneously, the one tending to make silver halide grains developable and the other tending to convert the developable grains into the original undevelopable state.

For exposure to light of uniform intensity that equation suggests that at low developable densities the rate of the forward reaction is slow, it increases as moderate developable densities are attained and again diminishes at the higher developable densities. It further postulates that initially the reverse reaction is slow but increases steadily as higher developable densities are reached, and at some point the rates of the forward and reverse reactions are equal. At this equilibrium point the maximum developable density has been attained. That equation also suggests that at maximum developable density the reactions do not cease, but no increase of density is obtained because of the statistical equilibrium. This paper will attempt to explain the phenomena of the Herschel effect on the basis of these assumptions.

If, after an initial exposure sufficient to produce considerable developable density, a photographic emulsion is subjected to a second exposure, usually to light of longer wave-length, the existing developable density is usually decreased. This is the Herschel effect. It should not be confused with the reversal due to excessive exposure, solarization. In the latter case, after the maximum developable density is attained, continued exposure to the same radiation lessens the developable density. The Herschel effect, however, assumes a decrease in developable density beginning simultaneously with exposure to the second radiation even though the maximum developable density for the first radiation has not been attained.

The most satisfactory method which we have found for producing the Herschel effect follows: A sheet of Azo No. 5 paper was exposed for three seconds to the unscreened radiation from a 100-watt lamp placed at a distance of 35 cm from the emulsion. Then, a Wratten safelight filter, series 00, was put in the place of the glass of an ordinary printing frame. Upon this, in the manner employed in making contact prints, was laid first, an opaque mask to shield the edges of the paper from a second exposure, and upon this was placed a negative, and then the previously exposed paper. A second exposure was made by placing this printing assembly at a distance of 35 cm from a 400-watt lamp and allowing the light from this source to pass through the filter and the negative and fall upon the paper for 13 minutes. The paper then was developed and fixed, yielding the negative image due to the Herschel effect shown in Fig. 2. The black borders in this print and the others show what the developable density would have been if there had been no bleaching by a second exposure. All of the photoengravings were made directly from the prints.

Jones and Hall[2] have shown that decreasing the intensity of the light to which an emulsion is exposed, reduces the maximum developable density obtainable. This can be explained upon the basis of a forward and reverse reaction if we assume that diminution of the intensity of the exposing light retards the rate of the reverse reaction, but retards that of the forward reaction to a greater degree, so that equilibrium is reached at a lower density.

Ross[3] has shown that the maximum developable density decreases with increasing wave-

[1] Blair and Leighton, J. Phys. Chem. **36**, 1649 (1932).

[2] Jones and Hall, Proc. Seventh Int. Cong. Phot. p. 115 (1929).

[3] F. E. Ross, *The Physics of the Developed Photographic Image*, Eastman Kodak Co., monograph No. 5.

length for all lengths greater than 4800A. The maximum developable density which can be obtained by yellow light is relatively low. If, therefore, a latent image of higher developable density is exposed to yellow light the developable density should be reduced to the equilibrium density for the yellow light. If a previously unexposed emulsion is given an optimum exposure to yellow light one should also expect to obtain the same equilibrium density.

In order to verify this prediction one-half (which will hereafter be designated as side *A*) of a sheet of Azo No. 1 paper was given an exposure of 20 seconds at a distance of 80 cm from an unscreened 60-watt lamp, the other half (side *B*) was not exposed. A strip containing both exposed and unexposed portions of this sheet was cut off and developed. This is strip "1" of Fig. 1. The whole of the remainder of the sheet was exposed to a safelight lamp containing a 16 c.p. bulb and a series 00 Wratten safelight at a distance of 45 cm for 14 days. At the end of that time a second strip was cut from the sheet, and finished as usual. This strip "2" in the same figure and it shows that a common density was obtained as predicted. Side *B* was again shielded and side *A* given another exposure to white light. The next strip, "3," was then cut off and developed. It shows that side *A* was again as dense as before the exposure to yellow light. Both sides were again exposed to yellow light for 14 days. Strip "4" shows that again a common equilibrium density had been reached on both sides. Side *A* was given another strong exposure to white light and strip "5" shows the effect.

We have obtained data which indicates that

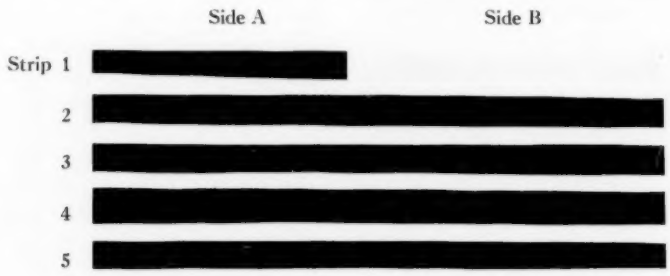


FIG. 1.

the reverse reaction is more rapid in intense light than in faint. This would suggest that when previously exposed paper is placed behind a negative and given a second exposure to light of long wave-length, the bleaching would be more rapid behind the clearer parts of the negative. This produces the Herschel effect shown in Fig. 2.

Insofar as the light is fainter behind the more dense parts of the negative, the equilibrium density there is lower than behind the clear parts of the negative. We would expect on the basis of our hypothesis that the resultant rate of bleaching would decrease as the density approaches the equilibrium density. This decrease in resultant rate has been observed and reported by Leszynski.[4]

Our hypothesis suggests that at some time, when the density behind the clearer parts of the negative is nearly to the equilibrium density, and the density behind the darker parts still diminishes because it approaches a lower level, that the

FIG. 2.

[4] Leszynski, Zeits. f. Wiss. Phot. **24**, 275 (1926).

densities of all parts of the paper would be about the same. This result was obtained by a second exposure of 25 minutes and is shown in Fig. 3. With flat negatives practically no contrast existed in prints made in this manner.

FIG 3.

After still longer exposure to the bleaching light we would expect on this basis that all of the emulsion would have attained the equilibrium density for the existing intensity of the impingent radiation. We would then expect that printing through a negative would yield a positive image. This effect was markedly obtained by exposing the paper to white light for 3 seconds as before and then to the yellow light for 2 hours. Fig. 4 shows a print obtained in this manner. The images of the trees shown in the prints were the clearer parts of the negative used. Our hypothesis would suggest that the images of the trees shown in Fig. 2. should be as dark or darker than those shown in Fig. 4. This is actually the case although the varying backgrounds at first suggest that the contrary was true. We have found no instance where prolonging the second exposure in order to obtain a positive image increased the developable density at any point.

Trivelli[5] and others have suggested that the Herschel effect is not due to a desensitization phenomenon, but is due to a destruction of the developability of the latent image. Our hypothesis suggests that this destruction is a consequence of the nature of the reaction forming the latent image. It explains why a second exposure to light of long wave-length decreases great developable densities, increases small developable densities, and has no effect on certain intermediate densities. In connection with the work of Ross it shows why the longer wave-lengths are more effective in producing the Herschel effect. In connection with the work of Jones and Hall it shows why the Herschel effect can be obtained with blue light only when the intensity of the blue light is very faint. It explains why prolonged exposure to light of longer wave-lengths converts a negative image into a positive image. Other characteristics of the Herschel effect can be explained by the same hypothesis.

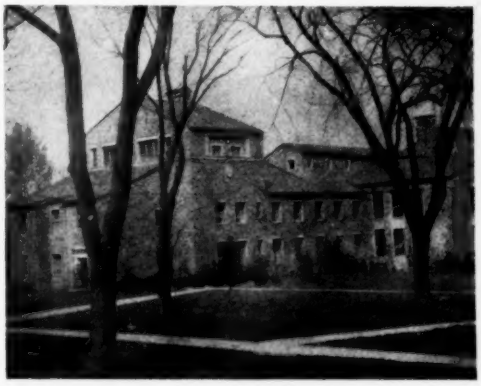


FIG. 4.

[5] Trivelli, J. Frank. Inst. **207**, 765 (1929).

The Relation between the Resolving Power of a Spectroscope and the Principle of Uncertainty

STANFORD GOLDMAN, *Harvard University*

(Received October 22, 1932)

THE resolving power of a spectroscope presents another interesting illustration of the principle of uncertainty. This has been implied in the works of many writers, but the importance of this illustration makes it worth while to carry through the formulation of the relationship in detail.

Let us consider the diffraction grating. Its resolving power is well known to be

$$\lambda/\Delta\lambda = \nu/\Delta\nu = mn \tag{1}$$

where $\Delta\lambda \equiv$ the difference in wave-length of two lines which the spectroscope can just revolve, $m \equiv$ the number of lines ruled on the grating, and $n \equiv$ the order of the spectrum. From the familiar formula for the grating

$$n\lambda = (A/m)\sin\theta \tag{2}$$

where A (Fig. 1) is the total length over which the grating is ruled. Then

$$mn = A\sin\theta/\lambda = B/\lambda. \tag{3}$$

Thus the resolving power of a grating is numerically equal to the number of waves in the train from Q to S, or in other words to the path difference in wave-lengths between the first and last interfering wavelets. A similar result holds for any kind of interference spectroscope. (Echelon, interferometers.)

Now the optical path $OQSP$ is longer than the path ORP by a length of time

$$\Delta t = B/c \tag{4}$$

where c is the velocity of light in air. Thus if the grating attains the resolving power given by Eq. (3), Δt is an indefiniteness in the time specification of light which is resolved by the spectroscope, which indefiniteness is introduced by the spectroscope.

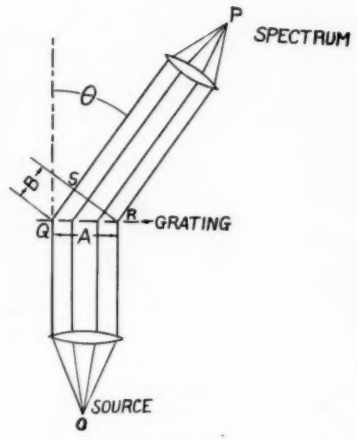


FIG. 1.

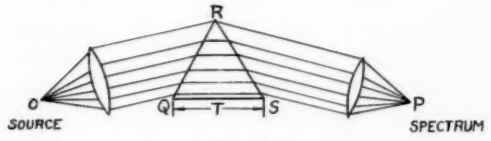


FIG. 2.

Consider next the indefiniteness in the energy of a photon resolved by the spectroscope. By Eqs. (1) and (3), we have

$$\Delta E = h\Delta\nu = h\nu/mn = h\nu(\lambda/B) = hc/B. \tag{5}$$

Thus

$$\Delta t \cdot \Delta E = h. \tag{6}$$

Consider next a prism spectroscope. Here we have the well-known value for the resolving power

$$\lambda/\Delta\lambda = \nu/\Delta\nu = -T(d\mu/d\lambda) \tag{7}$$

where $\mu \equiv$ refractive index, $T \equiv$ thickness of base of the prism, $\lambda \equiv$ wave-length in air. Then

$$\Delta E = h\Delta\nu = h\nu \frac{1}{-T\dfrac{d\mu}{d\lambda}} = \frac{h\nu}{T}\left[\frac{-1}{-\dfrac{\nu}{\lambda}\dfrac{d}{d\nu}\left(\dfrac{\lambda\nu}{a}\right)}\right] = \frac{h\nu}{T}\left\{\frac{-1}{-\dfrac{\nu}{\lambda}\left[\lambda\dfrac{d}{d\nu}\left(\dfrac{\nu}{a}\right)-\dfrac{\lambda}{a}\right]}\right\} = \frac{h}{T\left(\dfrac{1}{U}-\dfrac{1}{a}\right)}, \quad (8)^1$$

where $U \equiv [(d/d\nu)(\nu/a)]^{-1} \equiv$ the group velocity in prism, $a \equiv \lambda\nu/\mu \equiv$ the phase velocity in the prism. By Fermat's principle, the time taken to go from O to P is the same along all paths for an entity travelling with the phase velocity. However, a photon, being energy, must be considered as travelling with the group velocity. Therefore the difference in time along the optical paths of the extreme rays through the prism spectrometer, ORP and $OQSP$, is

$$\Delta t = T(1/U - 1/a). \quad (9)$$

This is the indefiniteness, introduced by the spectroscope, in the time specification of the photons resolved by the prism. Therefore, also in this case

$$\Delta E \cdot \Delta t = h. \quad (10)$$

[1] I have preferred to start with the form Eq. (7) for the resolving power because of its familiarity, rather than with any of the forms of Eq. (8) which may also be found in the literature.